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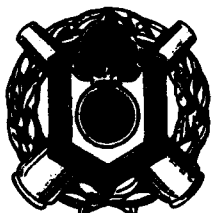
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THEORY OF ELECTROMAGNETIC SHIELDING FOR CONDUCTING CYLINDERS AND SPHERES

John Podesta

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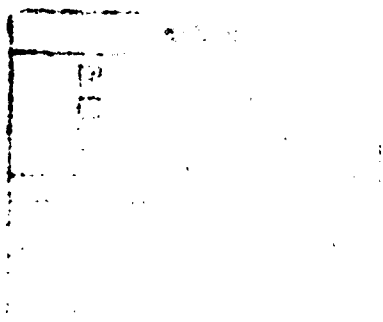
CONTENTS

	Page
Introduction	1
Theory: Perpendicular Incidence	4
TM Incident Wave	12
TE Incident Wave	16
Oblique Incidence	20
Oblique TM Wave	28
Oblique TE Wave	33
Shielding Effectiveness of the Cylinder	38
Electromagnetic Fields for the Sphere	44
Shielding Effectiveness of the Sphere	55
Numerical Calculations for the Sphere	59
Conclusions	65
References	106
Bibliography	109
Appendixes	
A Numerical Calculation of Bessel Functions	67
B Calculation of d_0	72
C Calculation of D_0	81
D Approximate Formulas for Shielding Effectiveness	86
E Alternate Definition of Shielding Effectiveness	90
F Approximations for the Spherical Bessel Functions	96
Distribution List	113

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FIGURES

	Page
1 Geometry for perpendicular incidence. Top view.	6
2 Geometry for oblique incidence. Side view.	25
3 Shielding effectiveness for the cylinder: TM incident wave.	41
4 Shielding effectiveness for the cylinder: TE incident wave.	43
5 Shielding effectiveness for the sphere: low frequency results.	60
6 Shielding effectiveness for the sphere: high frequency results	64



INTRODUCTION

Due to the serious interest in radiated electromagnetic effects which exists within the Department of Defense and the electromagnetic compatibility (EMC) community the subject of electromagnetic shielding has risen to a position of high importance. Military systems and support equipment are generally required to operate either during or after exposure to a variety of intense electromagnetic environments, such as electromagnetic pulse (EMP), electromagnetic interference (EMI), jammers, and directed energy weapons, that is, high power microwaves (HPM). The problem of shielding by circular conducting cylinders is especially important for the analysis of military systems due to the congruity, or similarity of this shape to the shapes of airplanes, missiles, and artillery projectiles. Rough engineering estimates of the shielding inherent in these types of military systems may be made by approximating these structures by ideal cylindrical conducting shells.

The propagation of an electromagnetic planewave in the presence of a planar conducting sheet, an infinitely long cylindrical conducting surface, and a spherical conducting surface are among the few problems which admit an exact analytical solution and have therefore been investigated in some detail. These solutions have been used to make calculations of the electromagnetic attenuation, or shielding effectiveness, of these structures. More complicated structures have also been investigated by various methods. The shielding effectiveness is the primary quantity of interest in the subject of electromagnetic shielding. It is a quantitative measure of the attenuation of incident electromagnetic energy due to the presence of a metallic surface or shield. It may be roughly defined as the intensity of the electromagnetic field at a point in space within the shielded volume to the intensity of the incident planewave.

The purpose of this report is to calculate the shielding effectiveness versus frequency for circular cylindrical shields and for spherical shields. An important result of this paper is the derivation of an approximate formula which expresses the shielding effectiveness as a function of the frequency, the conductivity, and the thickness of the shield. The results show that this formula is a good approximation for frequencies in the resonance regime. This approximate formula permits quick shielding calculations to be made without the need for computer programs or complicated mathematics.

Due to its geometrical and mathematical simplicity, the problem of scattering of a planewave by a solid circular cylinder at perpendicular incidence was solved as early as 1881 by Lord Rayleigh, (ref 1), and independently by von Ignatowski in 1905 (ref 2). Following these original works many German authors investigated this problem, including von Ignatowski in 1907, (ref 3), Seitz in 1905 and 1906, (refs 4-6), von Schaefer in

1907 (refs 7,8) and Debye in 1908 (ref 9). The scattering of a normally incident planewave by a solid cylinder with a concentric cylindrical sheath was first solved by Adey in 1956 (ref 10), Tang in 1957 (ref 11) and latter investigated by Kerker and Matijevic in 1961 (ref 12). The problem of scattering by a solid cylinder due to a planewave at oblique incidence was not solved for many years following the initial work of Lord Rayleigh and von Ignatowski. The full solution of this problem was given by Wait in 1955 (ref 13) although the method and form of the solution was given in the classic textbook on electromagnetic theory by Stratton in 1941 (ref 14). Exact solutions for scattering by cylinders due to other types of sources, such as line sources and dipole sources, may be found in the encyclopedic reference book on electromagnetic scattering by Bowman, Senior, and Uslenghi (ref 15).

The first analysis of the shielding properties of hollow conducting cylinders was made by Wu and Tsai in 1974 (ref 16). These authors solved the scattering problem for a cylindrical conducting shell of inner radius $r=a$ and outer radius $r=b$ due to a normally incident planewave, and then calculated the shielding effectiveness as a function of ka for a cylinder of radius $a=3.0$ mm and thickness $b-a=0.1$ mm. The calculation covered the frequency range from $ka=1.1$ to $ka=4.0$, and, therefore, was limited to a narrow band in the vicinity of the first resonant frequency. In a second paper, Wu and Tsai analyzed the shielding effectiveness for a planewave at oblique incidence (ref 17), and in a third paper they investigated the transient response of a cylindrical shield to simple EMP wave form (ref 18). These are the only results on the shielding problem for circular conducting cylinders which are known to this author. In this report some of this earlier work shall be extended. The shielding effectiveness shall be calculated over a wide range of frequencies which covers several orders of magnitude. In reference 16 no details of the numerical calculations were given other than a reference to a well known handbook of functions (ref 19). In this report, full details of the numerical calculations shall be included for completeness. This analysis leads to a derivation of the resonance condition and a derivation of an approximate formula for the shielding effectiveness in the resonance regime. No results for the shielding effectiveness are presented for the case of an obliquely incident planewave due to time constraints on this work.

In the second part of this study the shielding properties of a spherical shield are investigated. This problem is closely related to the problem for the cylinder and the results are very similar. The complete theory for scattering of an electromagnetic planewave by a sphere is usually attributed to Mie (ref 20). However, as pointed out by Kerker (ref 21) the theory had been worked out earlier by several authors, and even as early as 1863 by Clebsch. Some of the earlier work included the theory of scattering by a perfectly conducting sphere by Lorenz in 1890, the theory of scattering by a perfectly conducting sphere by J.J. Thomson in 1893, and an extension of Thomson's theory to dielectric and imperfectly conducting spheres by Love in 1899. Following the work of

Mie in 1908, further contributions to the theory were made by Debye in 1909 (ref 22). The most concise formulations of the solutions to the scattering problem for the sphere are in terms of spherical vector wavefunctions which comprise a complete set of solutions to the vector wave equation in spherical coordinates. Details of these solutions are given in the text by Stratton (ref 14). The problem of scattering by a spherical shell was first derived by Aden and Kerker in 1951 (ref 23). These authors consider a spherical scatterer consisting of two concentric homogeneous regions which are in direct contact: a spherical interior region covered with a concentric spherical shell. The general solutions are given for the case when the interior region and the concentric spherical shell have different complex dielectric constants and different complex propagation constants. The study by Aden and Kerker was extended by Wait in 1964 to the case when the scatterer consists of any number of concentric homogeneous regions (ref 24).

Stimulated by interest in nuclear electro-magnetic pulse (EMP), studies of the shielding of transient fields by a spherical metallic shield were conducted in 1965 by Harrison and Papas (ref 25). Using the steady state solutions for scattering by an incident planewave of frequency f (ref 14), the authors calculate the field amplitudes of the incident wave, that is, the shielding effectiveness, as a function of frequency. This result is just the transfer function for the field amplitudes, either electric or magnetic, at the center of the sphere due to an incident wave of frequency f . Using this transfer function and standard Fourier transform techniques, these authors make numerical calculations of the transient fields at the center of the sphere due to a Gaussian EMP wave form incident on the sphere. Further investigations of the steady state shielding problem were made by Chui, Dudley, and Bristol in 1969 (ref 26). These authors use the vector multipole expansions of the fields described in Jackson (ref 27) instead of Stratton's vector spherical wavefunctions. Mathematically, however, these two formulations are equivalent. The only other difference in their approach is that they consider a circularly polarized incident wave rather than a linearly polarized wave, which adds some symmetry to the problem. This work extends the results of Harrison and Papas by calculating the magnitudes of the electric and magnetic fields in the interior of the spherical shell along the z -axis, and not just at the center $z=0$. These calculations were performed at six frequencies, including the first two resonant frequencies. In addition, they present plots of the equi-field curves throughout the interior region for two different frequencies. The results of this work, and additional work by Dudley and Quintenz in 1975 (ref 28) show that the magnitudes of the fields away from the center of the sphere may be four orders of magnitude greater than the fields at the center.

The theory of the shielding effectiveness of a spherical conducting shell is developed in this report with reference to the fields at the center of the sphere. The approach used here is the same as that of Harrison and Papas (ref 25). Numerical calculations

are performed for a sphere with outer diameter $d=155$ mm and thickness $b-a=0.1$ mm, over a range of frequencies from 10^0 to 10^{10} Hz. The results show a series of discrete resonances at high-frequencies. Although these resonances are analogous to those for the cylinder problem, they have apparently not been thoroughly investigated in previous work. Equations for the resonance conditions are derived. In addition, an approximate formula for the shielding effectiveness is obtained which is accurate for frequencies in the resonance regime. To this author's knowledge this result has not been derived previously.

The contents of this report are organized as follows. The theory and results for the cylinder are presented in part 1, and the theory and results for the sphere are presented in part 2. In the first section, section 1, the field equations will be solved by reducing the Maxwell equations to a scalar wave equation, the scalar Helmholtz equation, in cylindrical coordinates. In section 2 and section 3 the exact solutions for the TM and TE fields shall be presented in the forms of series expansions of the fields and the expansion coefficients determined by the application of boundary conditions on the cylindrical surface. In sections 4, 5, and 6 the field equations shall be solved for the case of an obliquely incident planewave, that is, a planewave incident at an arbitrary angle to the z -axis. Then in section 7 the shielding effectiveness shall be defined and the results of numerical calculations of the shielding effectiveness versus frequency for the cylinder shall be presented for both the E-polarized and H-polarized incident waves. The numerical methods for evaluation the Bessel functions are described in Appendix A. The results are compared to the approximate formula for the shielding effectiveness which is derived in appendixes B, C, and D. An alternate definition of shielding effectiveness is defined in appendix E. In part 2, the Maxwell equations are solved by using the well known spherical vector wavefunctions which form a complete set of solutions to the vector wave equation, the vector Helmholtz equation, in spherical coordinates. In section 8 the scattering of a planewave by a spherical conducting shell is solved by expanding the electromagnetic fields in terms of spherical vector wavefunctions. The expansion coefficients are obtained by matching the boundary conditions at the spherical surface. In section 9 the field solutions are used to obtain expressions for the shielding effectiveness of the sphere, and in section 10 the numerical calculations of the shielding effectiveness are presented. The resonance conditions and the approximate formula for the shielding effectiveness are derived in appendix F. Finally, the conclusions are summarized in section 11.

THEORY: PERPENDICULAR INCIDENCE

An infinitely long cylindrical conducting shell of inner radius $r=a$ and outer radius $r=b$ has electrical conductivity σ , dielectric constant ϵ , and magnetic permeability μ . It is assumed that the conductor is a non-magnetic material, that is, it is a diamagnetic or

paramagnetic material, such as copper or aluminum, and therefore $\mu \approx \mu_0$. The case when the conductor is a ferromagnetic material is more complex due to the frequency and magnetic field dependence of the permeability, and therefore shall not be considered here. Consequently, this analysis does not rigorously apply to shielding materials such as iron or steel. It shall be assumed in the analysis that $\mu = \mu_0$.

The cylinder is located in empty space (vacuum) which has a dielectric constant ϵ_0 and permeability μ_0 . A linearly polarized planewave of frequency f is incident on the cylinder with its direction of propagation perpendicular to the cylinder axis. Two polarization states shall be analyzed separately: one state of linear polarization of the incident wave with the E-vector parallel to the cylinder axis (TM), and one polarization state with the H-vector parallel to the cylinder axis (TE). Any state of elliptical polarization of the incident planewave may be expressed, in general, as a sum of these two polarization states, TM and TE, with complex coefficients. Therefore, once solutions for the TM and TE incident waves are found, the general solution for an arbitrary state of polarization of the incident wave may be obtained by superposition of these independent solutions.

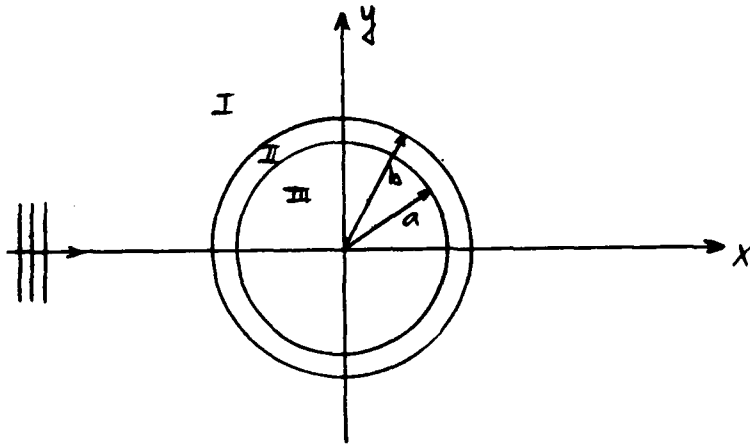


Figure 1. Geometry for perpendicular incidence. Top view.

The physical geometry for this problem is shown in figure 1. There are three distinct regions: I, II, and III, for which the electromagnetic fields must be determined. The boundary conditions on the field vectors at the interfaces of the different regions are as follows: the tangential E-fields and tangential H-fields must be continuous, and, the normal D-fields and normal B-fields must be continuous. The electromagnetic field equations, the Maxwell equations, in MKSA units for a general conducting medium are as follows:

$$\nabla \times \mathbf{E} = - \frac{d\mathbf{B}}{dt},$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{d\mathbf{D}}{dt},$$

$$\nabla \cdot \mathbf{D} = 0,$$

$$\nabla \cdot \mathbf{B} = 0,$$

where the material equations are

$$\mathbf{D} = \epsilon \mathbf{E},$$

$$\mathbf{B} = \mu \mathbf{H},$$

$$\mathbf{J} = \sigma \mathbf{E}.$$

Since the incident planewave has a harmonic time dependence, this implies that all the fields shall have the same time dependence.

Therefore it is assumed that the field vectors E , H , D , and B vary in time as $e^{-i\omega t}$. Using this fact together with the constitutive equations, the Maxwell equations take the form

$$\nabla \times E = i\omega\mu_0 H, \quad (1)$$

$$\nabla \times H = (\sigma - i\omega\epsilon)E, \quad (2)$$

$$\nabla \cdot E = 0, \quad (3)$$

$$\nabla \cdot H = 0. \quad (4)$$

The wave equations for the E and H -fields may be derived from the field equations. Taking the curl of equation 1 and using the vector identity

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E,$$

and then substituting equations 2 and 3 into the result yields the vector wave equation, or Helmholtz equation,

$$(\nabla^2 + \gamma^2)E = 0, \quad (5)$$

where

$$\gamma^2 = i\omega\mu_0(\sigma - i\omega\epsilon).$$

Similarly, by taking the curl of equation 2, and then substituting equations 1 and 4, the wave equation for the H -field takes the same form:

$$(\nabla^2 + \gamma^2)H = 0, \quad (6)$$

where

$$\gamma^2 = i\omega\mu_0(\sigma - i\omega\epsilon). \quad (7)$$

In the case when the conductivity is zero, that is, in regions I and III, the same wave equation holds with $\sigma=0$, and $\epsilon=\epsilon_0$, which implies $\gamma=k$. As a mathematical note it should be mentioned that the wave equations, equations 5 and 6, are automatically

satisfied by the E and H-fields if the fields satisfy Maxwell's equations. That is, If E and H satisfy Maxwell's equations then they satisfy 5 and 6. However, if the fields satisfy 5 and 6 this does not imply that they satisfy the Maxwell equations. Therefore, once solutions of the wave equations are obtained it is necessary to verify that they satisfy the Maxwell equations.

In the first case to be considered the incident E-field is polarized in the z-direction:

$$E_z^{inc} = E_0 e^{ikx}.$$

Symmetry about the z-axis implies that the total E-field in each of the three regions is also polarized in the z-direction. Thus the only nonzero component of the E-field is E_z in the TM case.

By using the first Maxwell equation, equation 1, and the expression for the curl operator in cylindrical coordinates, the total H-field has two nonzero components given by

$$H_r = \frac{1}{i\omega\mu_0} \frac{1}{r} \frac{dE_z}{d\phi}, \quad (8)$$

$$H_\phi = \frac{-1}{i\omega\mu_0} \frac{dE_z}{dr}. \quad (9)$$

Thus, when the electric field component E_z is known the magnetic fields may be obtained by differentiation. The electric field E_z will be obtained in each of the three regions I, II, and III by solving the respective wave equations. The magnetic fields will be calculated from E_z by using equations 8 and 9. By using this approach Maxwell's equations are automatically satisfied. Similarly, in the TE case the only nonzero component of the H-field is H_z . Using the second Maxwell equation, equation 2, the components of the E-field in this case are given by

$$E_r = \frac{1}{\sigma - i\omega\epsilon} \frac{1}{r} \frac{dH_z}{d\phi}, \quad (10)$$

$$E_\phi = \frac{-1}{\sigma - i\omega\epsilon} \frac{dH_z}{dr}. \quad (11)$$

The material parameters in region I are μ_0 , ϵ_0 , and $\sigma=0$. The wave equation for E_z (TM case) or H_z (TE case) is given by

$$(\nabla^2 + k^2)\psi = 0, \quad k^2 = \frac{\omega^2}{c^2};$$

where ψ is equal to E_z or H_z . Using the expression for the Laplacian in cylindrical coordinates this becomes

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\psi}{dr} \right) + \frac{1}{r^2} \frac{d^2\psi}{d\phi^2} + k^2\psi = 0. \quad (12)$$

To solve this equation use separation of variables. Let $\psi=R(r)Q(\phi)$. Then the equation for the angle variable becomes

$$\frac{d^2Q}{d\phi^2} + n^2Q = 0, \quad (13)$$

which has solution

$$Q(\phi) = e^{\pm i n \phi}.$$

The parameter n^2 is the separation constant. To ensure that the solution is single valued n must be an integer: $n=0, \pm 1, \pm 2, \dots$. The radial equation takes the form

$$\frac{d^2R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left(k^2 + \frac{n^2}{r^2} \right) R = 0. \quad (14)$$

If the substitution $x=kr$ is made, then this equation takes the standard form of Bessel's differential equation:

$$\frac{d^2R}{dx^2} + \frac{1}{x} \frac{dR}{dx} + \left(1 - \frac{n^2}{x^2} \right) R = 0. \quad (15)$$

The linearly independent solutions are the Bessel functions of the first and second kinds

$$R(x) = J_n(x), \quad Y_n(x).$$

Another set of linearly independent solutions are the Hankel functions

$$H_n^1(x) = J_n(x) + iY_n(x),$$

$$H_n^2(x) = J_n(x) - iY_n(x).$$

The total field in region I may be written as a superposition of the incident and scattered fields in the form

$$\psi = e^{ikx} + \sum_{n=-\infty}^{\infty} i^n a_n H_n^1(kr) e^{in\phi}. \quad (16)$$

The Hankel function $H^1(kr)$ is chosen since it represents an outgoing wave for large r . The incident field may be expanded in a Fourier series as follows

$$e^{ikx} = e^{ikr \cos(\phi)} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\phi}. \quad (17)$$

Therefore, using this expansion the total field may be written

$$\psi = \sum_{n=-\infty}^{\infty} i^n [J_n(kr) + a_n H_n^1(kr)] e^{in\phi}, \quad r > a. \quad (18)$$

Next, solutions shall be found for the wave equation in region II. The material parameters in region II are μ_0 , ϵ , and σ . The wave equation for E_z in the TM case, or H_z in the TE case, is

$$(\nabla^2 + \gamma^2)\psi = 0,$$

where, from 7

$$\gamma^2 = \frac{\omega^2}{c^2} \left(\frac{\epsilon}{\epsilon_0} + i \frac{\sigma}{\omega \epsilon_0} \right). \quad (19)$$

As in the previous derivation, separation of variables leads to the solution

$$\psi = \sum_{n=-\infty}^{\infty} i^n [b_n J_n(\gamma r) + c_n H_n^1(\gamma r)] e^{in\phi}, \quad a < r < b. \quad (20)$$

The combination of functions $J_n(\gamma r)$ and $H_n^1(\gamma r)$ are chosen so the solution in this region has a similar form to the solution in region I. The arguments of these functions are now complex, however, since γ is complex. For a good conductor, and for frequencies such that $(\sigma/\omega\epsilon_0) \gg 1$, approximate relations for γ are, from 19,

$$\gamma^2 = \frac{\omega^2}{c^2} \left(i \frac{\sigma}{\omega\epsilon_0} \right), \quad \text{and} \quad \gamma = \frac{1}{c} \left(\frac{\omega\sigma}{\epsilon_0} \right)^{1/2} e^{i\frac{\pi}{4}} = (\omega\sigma\mu_0)^{1/2} e^{i\frac{\pi}{4}}. \quad (21)$$

In region III the material parameters are μ_0 , ϵ_0 , and $\sigma=0$. Thus the wave equation has the same form as in region I:

$$(\nabla^2 + k^2)\psi = 0, \quad k^2 = \frac{\omega^2}{c^2}.$$

As before, separation of variables yields the solution

$$\psi = \sum_{n=-\infty}^{\infty} i^n d_n J_n(kr) e^{in\phi}, \quad r < a. \quad (22)$$

Only the Bessel function $J_n(kr)$ is used since the other solutions, which contain Neumann functions, approach infinity as r approaches zero. These other solutions are physically unacceptable.

Due to the fact that only the Hankel function $H_n^1(x)$ shall be used, the superscript $(^1)$ shall be omitted hereafter. Therefore, whenever the function $H_n(x)$ appears it should be understood that it stands for $H_n^1(x)$. The reason for this is to avoid confusion with the prime symbol $(')$ which is used for differentiation. This convention shall be adhered to throughout this thesis.

2. TM INCIDENT WAVE

The full solutions for the fields in the TM case are obtained by using equations 20, 21, and 22 which give solutions for E_z and then using equations 8 and 9 to determine H_r and H_ϕ . The full electromagnetic field solutions are, in region I,

$$E_z = E_0 \sum_{n=-\infty}^{\infty} i^n [J_n(kr) + a_n H_n(kr)] e^{in\phi}, \quad r > b; \quad (23)$$

$$H_r = \frac{E_0}{i\omega\mu_0} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} [J_n(kr) + a_n H_n(kr)] e^{in\phi}, \quad r > b; \quad (24)$$

$$H_\phi = \frac{-E_0 k}{i\omega\mu_0} \sum_{n=-\infty}^{\infty} i^n [J'_n(kr) + a_n H'_n(kr)] e^{in\phi}, \quad r > b; \quad (25)$$

region II:

$$E_z = E_0 \sum_{n=-\infty}^{\infty} i^n [b_n J_n(\gamma r) + c_n H_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (26)$$

$$H_r = \frac{E_0}{i\omega\mu_0} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} [b_n J_n(\gamma r) + c_n H_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (27)$$

$$H_\phi = \frac{-E_0 \gamma}{i\omega\mu_0} \sum_{n=-\infty}^{\infty} i^n [b_n J'_n(\gamma r) + c_n H'_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (28)$$

region III:

$$E_z = E_0 \sum_{n=-\infty}^{\infty} i^n d_n J_n(kr) e^{in\phi}, \quad r < a; \quad (29)$$

$$H_r = \frac{E_0}{i\omega\mu_0} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} d_n J_n(kr) e^{in\phi}, \quad r < a; \quad (30)$$

$$H_\phi = \frac{-k E_0}{i\omega\mu_0} \sum_{n=-\infty}^{\infty} i^n d_n J'_n(kr) e^{in\phi}, \quad r < a. \quad (31)$$

The coefficients a_n , b_n , c_n , and d_n are determined by matching boundary conditions at the interfaces. Continuity of the tangential E-field at $r=b$ implies

$$J_n(kb) + a_n H_n(kb) = b_n J_n(\gamma b) + c_n H_n(\gamma b). \quad (32)$$

Continuity of the tangential E-field at $r=a$ implies

$$b_n J_n(\gamma a) + c_n H_n(\gamma a) = d_n J_n(ka). \quad (33)$$

Continuity of the tangential H-field at $r=b$ implies

$$k[J'_n(kb) + a_n H'_n(kb)] = \gamma[b_n J'_n(\gamma b) + c_n H'_n(\gamma b)]. \quad (34)$$

Continuity of the tangential H-field at $r=a$ implies

$$\gamma[b_n J'_n(\gamma a) + c_n H'_n(\gamma a)] = k d_n J'_n(ka). \quad (35)$$

These are all the required boundary conditions. Since $\mu=\mu_0$, the boundary condition on the normal component of B is redundant. These equations; 32, 33, 34, and 35; comprise a nonhomogeneous system of four equations in the four unknowns a_n , b_n , c_n , and d_n . The solutions of this system are obtained directly with the use of Kramer's rule. This results are given by the relations

$$a_n = \frac{U_1}{V}, \quad b_n = \frac{U_2}{V}, \quad c_n = \frac{U_3}{V}, \quad d_n = \frac{U_4}{V}; \quad (36)$$

where the U_i and V are determinants given as follows:

$$U_1 = \begin{vmatrix} -J_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & 0 \\ 0 & J_n(\gamma a) & H_n(\gamma a) & -J_n(ka) \\ -kJ'_n(kb) & -\gamma J'_n(\gamma b) & -\gamma H'_n(\gamma b) & 0 \\ 0 & \gamma J'_n(\gamma a) & \gamma H'_n(\gamma a) & -kJ'_n(ka) \end{vmatrix}, \quad (37)$$

$$U_2 = \begin{vmatrix} H_n(kb) & -J_n(kb) & -H_n(\gamma b) & 0 \\ 0 & 0 & H_n(\gamma a) & -J_n(ka) \\ kH'_n(kb) & -kJ'_n(kb) & -\gamma H'_n(\gamma b) & 0 \\ 0 & 0 & \gamma H'_n(\gamma a) & -kJ'_n(ka) \end{vmatrix}, \quad (38)$$

$$U_3 = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -J_n(kb) & 0 \\ 0 & J_n(\gamma a) & 0 & -J_n(ka) \\ kH'_n(kb) & -\gamma J'_n(\gamma b) & -kJ'_n(kb) & 0 \\ 0 & \gamma J'_n(\gamma a) & 0 & -kJ'_n(ka) \end{vmatrix}, \quad (39)$$

$$U_4 = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & -J_n(kb) \\ 0 & J_n(\gamma a) & H_n(\gamma a) & 0 \\ kH'_n(kb) & -\gamma J'_n(\gamma b) & -\gamma H'_n(\gamma b) & -kJ'_n(kb) \\ 0 & \gamma J'_n(\gamma a) & \gamma H'_n(\gamma a) & 0 \end{vmatrix}, \quad (40)$$

and

$$V = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & 0 \\ 0 & J_n(\gamma a) & H_n(\gamma a) & -J_n(ka) \\ kH'_n(kb) & -\gamma J'_n(\gamma b) & -\gamma H'_n(\gamma b) & 0 \\ 0 & \gamma J'_n(\gamma a) & \gamma H'_n(\gamma a) & -kJ'_n(ka) \end{vmatrix}. \quad (41)$$

This completes the solutions for the fields in the TM case. The expansion coefficients may be determined explicitly by evaluating the above determinants.

Although the series solutions of the fields are rather cumbersome since the expansion coefficients are given by quotients of determinants of Bessel and Hankel functions, they do constitute exact solutions, and they are sufficient for the purposes of this study. It is very desirable, and it seems very reasonable to expect that the field solutions may be expressible in a closed form in terms of an integral. This author is not aware if such a closed form integral solution exists or has been derived previously. This is a point of departure for further investigations.

3. TE INCIDENT WAVE

The full solutions for the fields in the TE case are obtained by using equations 20, 21, and 22 which give solutions for H_z and then using equations 10 and 11 to determine E_r and E_ϕ . The full electromagnetic field solutions are, in region I:

$$H_z = H_0 \sum_{n=-\infty}^{\infty} i^n [J_n(kr) + A_n H_n(kr)] e^{in\phi}, \quad r > b; \quad (42)$$

$$E_r = \frac{-H_0}{i\omega\epsilon_0} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} [J_n(kr) + A_n H_n(kr)] e^{in\phi}, \quad r > b; \quad (43)$$

$$E_\phi = \frac{H_0 k}{i\omega\epsilon_0} \sum_{n=-\infty}^{\infty} i^n [J'_n(kr) + A_n H'_n(kr)] e^{in\phi}, \quad r > b; \quad (43)$$

region II:

$$H_z = H_0 \sum_{n=-\infty}^{\infty} i^n [B_n J_n(\gamma r) + C_n H_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (44)$$

$$E_r = \frac{H_0}{\sigma - i\omega\epsilon} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} [B_n J_n(\gamma r) + C_n H_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (45)$$

$$E_\phi = \frac{-H_0 \gamma}{\sigma - i\omega\epsilon} \sum_{n=-\infty}^{\infty} i^n [B_n J'_n(\gamma r) + C_n H'_n(\gamma r)] e^{in\phi}, \quad a < r < b; \quad (46)$$

region III:

$$H_z = H_0 \sum_{n=-\infty}^{\infty} i^n D_n J_n(kr) e^{in\phi}, \quad r < a; \quad (47)$$

$$E_r = \frac{-H_0}{i\omega\epsilon_0} \frac{1}{r} \sum_{n=-\infty}^{\infty} n i^{n+1} D_n J_n(kr) e^{in\phi}, \quad r < a; \quad (48)$$

$$E_\phi = \frac{H_0 k}{i\omega\epsilon_0} \sum_{n=-\infty}^{\infty} i^n D_n J'_n(kr) e^{in\phi}, \quad r < a. \quad (49)$$

As in the TM case, coefficients A_n , B_n , C_n , and D_n are determined by matching boundary conditions at the interfaces. Continuity of the tangential H-field at $r=b$ implies

$$J_n(kb) + A_n H_n(kb) = B_n J_n(\gamma b) + C_n H_n(\gamma b). \quad (50)$$

Continuity of the tangential H-field at $r=a$ implies

$$B_n J_n(\gamma a) + C_n H_n(\gamma a) = D_n J_n(ka). \quad (51)$$

Continuity of the tangential E-field at $r=b$ implies

$$\frac{k}{i\omega\epsilon_0} [J'_n(kb) + A_n H'_n(kb)] = \frac{-\gamma}{\sigma - i\omega\epsilon} [B_n J'_n(\gamma b) + C_n H'_n(\gamma b)]. \quad (52)$$

Continuity of the tangential E-field at $r=a$ implies

$$\frac{-\gamma}{\sigma - i\omega\epsilon} [B_n J'_n(\gamma a) + C_n H'_n(\gamma a)] = \frac{k}{i\omega\epsilon_0} D_n J'_n(ka). \quad (53)$$

These are all the required boundary conditions. These equations; 50, 51, 52, and 53; comprise a nonhomogeneous system of four equations in the four unknowns A_n , B_n , C_n , and D_n . As before, the solutions are obtained with the use of Kramer's rule. The results are as follows:

$$A_n = \frac{X_1}{Y}, \quad B_n = \frac{X_2}{Y}, \quad C_n = \frac{X_3}{Y}, \quad D_n = \frac{X_4}{Y}; \quad (54)$$

where the X_i and Y are determinants given as follows:

$$X_1 = \begin{vmatrix} -J_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & 0 \\ 0 & J_n(\gamma a) & H_n(\gamma a) & -J_n(ka) \\ \frac{-k}{i\omega\epsilon_0}J'_n(kb) & \frac{\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma b) & \frac{\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma b) & 0 \\ 0 & \frac{-\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma a) & \frac{-\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma a) & \frac{-k}{i\omega\epsilon_0}J'_n(ka) \end{vmatrix}, \quad (55)$$

$$X_2 = \begin{vmatrix} H_n(kb) & -J_n(kb) & -H_n(\gamma b) & 0 \\ 0 & 0 & H_n(\gamma a) & -J_n(ka) \\ \frac{k}{i\omega\epsilon_0}H'_n(kb) & \frac{-k}{i\omega\epsilon_0}J'_n(kb) & \frac{\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma b) & 0 \\ 0 & 0 & \frac{-\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma a) & \frac{-k}{i\omega\epsilon_0}J'_n(ka) \end{vmatrix}, \quad (56)$$

$$X_3 = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -J_n(kb) & 0 \\ 0 & J_n(\gamma a) & 0 & -J_n(ka) \\ \frac{k}{i\omega\epsilon_0}H'_n(kb) & \frac{\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma b) & \frac{-k}{i\omega\epsilon_0}J'_n(kb) & 0 \\ 0 & \frac{-\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma a) & 0 & \frac{-k}{i\omega\epsilon_0}J'_n(ka) \end{vmatrix}, \quad (57)$$

$$X_4 = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & -J_n(kb) \\ 0 & J_n(\gamma a) & H_n(\gamma a) & 0 \\ \frac{k}{i\omega\epsilon_0}H'_n(kb) & \frac{\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma b) & \frac{\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma b) & \frac{-k}{i\omega\epsilon_0}J'_n(kb) \\ 0 & \frac{-\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma a) & \frac{-\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma a) & 0 \end{vmatrix}, \quad (58)$$

and

$$V = \begin{vmatrix} H_n(kb) & -J_n(\gamma b) & -H_n(\gamma b) & 0 \\ 0 & J_n(\gamma a) & H_n(\gamma a) & -J_n(ka) \\ \frac{k}{i\omega\epsilon_0}H'_n(kb) & \frac{\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma b) & \frac{\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma b) & 0 \\ 0 & \frac{-\gamma}{\sigma-i\omega\epsilon}J'_n(\gamma a) & \frac{-\gamma}{\sigma-i\omega\epsilon}H'_n(\gamma a) & \frac{-k}{i\omega\epsilon_0}J'_n(ka) \end{vmatrix}. \quad (59)$$

This completes the solutions for the fields in the TE case. The expansion coefficients may be determined explicitly by evaluating the above determinants.

This concludes the presentation of the electromagnetic field solutions for the scattering of a normally incident planewave from a circular cylindrical conducting shell of inner radius $r=a$ and outer radius $r=b$. The expansion coefficients of the fields in the region $r < a$ shall be evaluated in the Appendixes for the purposes of calculating the shielding effectiveness. In the next sections, sections 4, 5, and 6 the solutions for scattering by an obliquely incident planewave shall be derived.

4. OBLIQUE INCIDENCE

In general, Maxwell's equations may be solved by the use of two auxiliary vectors Π_1 and Π_2 , called Hertz vectors. The time harmonic Maxwell equations, equations 1, 2, 3, and 4, are automatically satisfied if

$$\mathbf{E} = \nabla \times \nabla \times \Pi_1 + i\omega\mu \nabla \times \Pi_2, \quad (60)$$

$$\mathbf{H} = (\sigma - i\omega\epsilon) \nabla \times \Pi_1 + \nabla \times \nabla \times \Pi_2, \quad (61)$$

and if the Hertz vectors satisfy the vector wave equations

$$\nabla \times \nabla \times \Pi_1 - \nabla(\nabla \cdot \Pi_1) - \gamma^2 \Pi_1 = 0, \quad (62)$$

$$\nabla \times \nabla \times \Pi_2 - \nabla(\nabla \cdot \Pi_2) - \gamma^2 \Pi_2 = 0, \quad (63)$$

where, as before,

$$\gamma^2 = i\omega\mu_0(\sigma - i\omega\epsilon). \quad (64)$$

This may be verified by inserting equations 60 and 61 into equations 1 and 2, and then using equations 62 and 63. In the case of cylindrical symmetry the only nonzero components of the Hertz vectors are the z -components. If $\Pi_1 = \Psi_1 \mathbf{e}_z$ and $\Pi_2 = 0$, where Ψ_1 is a scalar function, then the corresponding solutions of Maxwell's equations are given by

$$\mathbf{E} = \nabla \times \nabla \times \Pi_1 = ih \frac{d\Psi_1}{dr} \mathbf{e}_r + ih \frac{1}{r} \frac{d\Psi_1}{d\phi} \mathbf{e}_\phi + (\gamma^2 - h^2) \Psi_1 \mathbf{e}_z, \quad (65)$$

$$\mathbf{H} = (\sigma - i\omega\epsilon) \nabla \times \Pi_1 = (\sigma - i\omega\epsilon) \left(\frac{1}{r} \frac{d\Psi_1}{d\phi} \mathbf{e}_r - \frac{d\Psi_1}{dr} \mathbf{e}_\phi \right), \quad (66)$$

where it is assumed that $\Psi_1 = \psi_1(r, \phi) e^{ihz}$. A z -dependence of the form e^{ihz} is chosen to match the form of the incident wave. The vectors \mathbf{e}_r , \mathbf{e}_ϕ , and \mathbf{e}_z are the usual unit vectors in the

cylindrical coordinate system. An independent set of solutions of Maxwell's equations is obtained if $\Pi_1=0$ and $\Pi_2=\Psi_2 e_z$ which yields

$$\mathbf{E} = i\omega\mu \nabla \times \Pi_2 = i\omega\mu \left(\frac{1}{r} \frac{d\Psi_2}{d\phi} \mathbf{e}_r - \frac{d\Psi_2}{dr} \mathbf{e}_\phi \right), \quad (67)$$

$$\mathbf{H} = \nabla \times \nabla \times \Pi_2 = ih \frac{d\Psi_2}{dr} \mathbf{e}_r + ih \frac{1}{r} \frac{d\Psi_2}{d\phi} \mathbf{e}_\phi + (\gamma^2 - h^2) \Psi_2 \mathbf{e}_z, \quad (68)$$

where $\Psi_2 = \psi_2(r, \phi) e^{ihz}$. Note that the solutions based on the wavefunction Ψ_1 have a nonzero E_z component, and the solutions based on the wavefunction Ψ_2 have a nonzero H_z component.

Therefore, equations 65 and 66 represent TM fields and equations 67 and 68 represent TE fields.

For the case of perpendicular incidence a TM incident wave gives rise to a TM scattered wave and a TE incident wave gives rise to a TE scattered wave. This is not true in the case of oblique incidence. For oblique incidence a pure TM incident wave gives rise to a combination of TM and TE scattered waves. And, similarly, a pure TE incident wave gives rise to a combination of TE and TM scattered waves. Therefore both TM and TE components of the scattered fields are required to express the total fields in each of the regions I, II, and III. This implies that both wavefunctions Ψ_1 and Ψ_2 must be used to give independent TM and TE solutions for the scattered fields, respectively.

In region I, $r > b$, the material parameters are ϵ_0 , μ_0 , and $\sigma=0$. The scalar Helmholtz equation takes the form

$$(\nabla^2 + k^2) \Psi = 0, \quad k^2 = \frac{\omega^2}{c^2}. \quad (69)$$

The solutions for the wavefunctions are

$$(TM) \quad \Psi_1 = \frac{E_0}{k\ell} e^{ihz} \sum_{-\infty}^{\infty} i^n a_n H_n(\ell r) e^{in\phi}, \quad (70)$$

$$(TE) \quad \Psi_2 = \frac{H_0}{k\ell} e^{ihz} \sum_{-\infty}^{\infty} i^n b_n H_n(\ell r) e^{in\phi}, \quad (71)$$

where $k^2 = k^2 - h^2$. The Hankel function is chosen so that the scattered field reduces to an outgoing wave as r approaches infinity. The constants E_0/k and H_0/k are included so that the expansion coefficients are dimensionless. For convenience, the field strengths E_0 and H_0 will be chosen to be equal to those of the incident planewave. In region II, $a < r < b$, the region within the conductor, the material parameters are ϵ , μ , and σ . The Helmholtz equation takes the form

$$(\nabla^2 + \gamma^2)\Psi = 0, \quad \gamma^2 = i\omega\mu(\sigma - i\omega\epsilon).$$

The solutions for the wavefunctions in region II are

$$(TM) \quad \Psi_1 = \frac{E_0}{k\chi} e^{ihz} \sum_{-\infty}^{\infty} i^n [c_n J_n(\chi r) + d_n H_n(\chi r)] e^{in\phi}, \quad (72)$$

$$(TE) \quad \Psi_2 = \frac{H_0}{k\chi} e^{ihz} \sum_{-\infty}^{\infty} i^n [f_n J_n(\chi r) + g_n H_n(\chi r)] e^{in\phi}, \quad (73)$$

where $\chi^2 = \gamma^2 - h^2$. Both Bessel and Hankel functions are required in the expansions 72 and 73 in order to match the interior and exterior fields. In region III, $r < a$, the material parameters are ϵ_0 and μ_0 . Therefore, the wavefunctions in the interior region, region III, take the forms

$$(TM) \quad \Psi_1 = \frac{E_0}{k\ell} e^{ihz} \sum_{-\infty}^{\infty} i^n p_n J_n(\ell r) e^{in\phi}, \quad (74)$$

$$(TE) \quad \Psi_2 = \frac{H_0}{k\ell} e^{ihz} \sum_{-\infty}^{\infty} i^n q_n J_n(\ell r) e^{in\phi}. \quad (75)$$

Only the Bessel function is required since it is finite at $r=0$. Using these results for the wavefunctions the scattered fields in a given region may be obtained by using equations 65, 66, 67, and 68. For example, in the exterior region, region I, where $r > b$, the TM scattered fields are obtained by inserting the wavefunction 70 into the set of equations 65 and 66, and the TE scattered fields

are obtained by inserting 71 into equations 67 and 68. The resulting TM scattered fields are, for $r > b$,

$$E_r^{TM} = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{h}{k} a_n H'_n(lr) e^{in\phi},$$

$$E_\phi^{TM} = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{h}{k l r} (in) a_n H_n(lr) e^{in\phi},$$

$$E_z^{TM} = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{l}{k} a_n H_n(lr) e^{in\phi},$$

$$H_r^{TM} = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{in}{l r} a_n H_n(lr) e^{in\phi},$$

$$H_\phi^{TM} = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n a_n H'_n(lr) e^{in\phi}.$$

And, the resulting TE scattered fields in the region $r > b$ are

$$E_r^{TE} = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{in}{l r} b_n H_n(lr) e^{in\phi},$$

$$E_\phi^{TE} = -iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n b_n H'_n(lr) e^{in\phi},$$

$$H_r^{TE} = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{h}{k} b_n H'_n(lr) e^{in\phi},$$

$$H_\phi^{TE} = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{h}{k l r} (in) b_n H_n(lr) e^{in\phi},$$

$$H_z^{TE} = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{l}{k} b_n H_n(lr) e^{in\phi}.$$

The scattered fields in the other regions are obtained similarly. In general, the scattered fields are of the form

$$E_s = E_s^{TM} + E_s^{TE}.$$

Having determined explicit expressions for the scattered fields the next step is to obtain expressions for the incident fields. The incident planewave propagates at an angle θ to the x-axis as shown in Figure 2. The k-vector, or wave normal, lies in the x-z plane. Two distinct states of linear polarization of the incident wave will be treated separately. The case in which the E-vector of the incident wave lies in the x-z plane shall be called the TM incident wave. And the case in which the H-vector of the incident wave lies in the x-z plane shall be called the TE incident wave. In rectangular coordinates the electromagnetic fields of the TM incident wave are given by

$$(TM) \quad E_{inc} = E_0(-\sin\theta e_x + \cos\theta e_z) e^{ik(x\cos\theta + z\sin\theta)},$$

$$(TM) \quad H_{inc} = -H_0 e_y e^{ik(x\cos\theta + z\sin\theta)}.$$

And the fields of the TE incident wave are

$$(TE) \quad E_{inc} = E_0 e_y e^{ik(x\cos\theta + z\sin\theta)},$$

$$(TE) \quad H_{inc} = H_0(-\sin\theta e_x + \cos\theta e_z) e^{ik(x\cos\theta + z\sin\theta)},$$

where $H_0 = \sqrt{\epsilon_0/\mu_0} E_0$, and $k = \omega/c$. The components of the incident wave in cylindrical coordinates may be determined by using the transformation of basis vectors:

$$\begin{aligned} e_r &= \cos\phi e_x + \sin\phi e_y \\ e_\phi &= -\sin\phi e_x + \cos\phi e_y. \end{aligned}$$

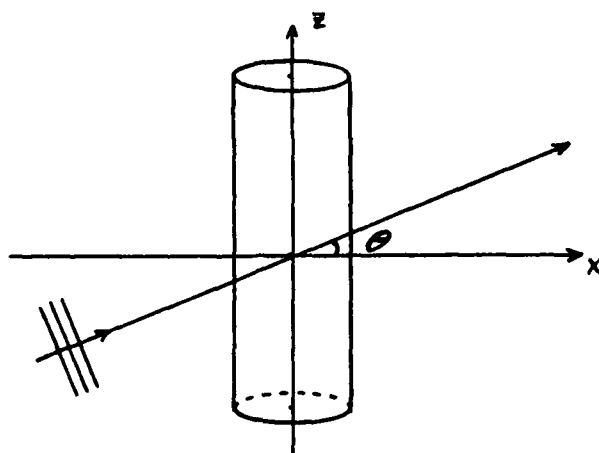


Figure 2. Geometry for oblique incidence. Side view.

The cylindrical components of the TM incident wave are

$$(TM) \quad E_r = E_0(-\sin\theta\cos\phi)e^{ihz}e^{ikx\cos\theta},$$

$$E_\phi = E_0(\sin\theta\sin\phi)e^{ihz}e^{ikx\cos\theta},$$

$$E_z = E_0(\cos\theta)e^{ihz}e^{ikx\cos\theta},$$

$$H_r = H_0(-\sin\phi)e^{ihz}e^{ikx\cos\theta},$$

$$H_\phi = H_0(-\cos\phi)e^{ihz}e^{ikx\cos\theta},$$

where $h=k\sin\theta$. Similarly, the cylindrical components of the TE incident wave are

$$(TE) \quad E_r = E_0(\sin\phi)e^{ihz}e^{ikx\cos\theta},$$

$$E_\phi = E_0(\cos\phi)e^{ihz}e^{ikx\cos\theta},$$

$$H_r = H_0(-\sin\theta\cos\phi)e^{ihz}e^{ikx\cos\theta},$$

$$H_\phi = H_0(\sin\theta\sin\phi)e^{ihz}e^{ikx\cos\theta},$$

$$H_z = H_0(\cos\theta)e^{ihz}e^{ikx\cos\theta}.$$

The incident fields may be expanded in a series of Bessel functions by using the Fourier series representation

$$e^{ikr\cos\phi\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(lr)e^{in\phi},$$

where $l=k\cos\theta$. The following identities are also needed

$$\sin\phi e^{ikr\cos\phi\cos\theta} = -\frac{1}{ilr} \frac{d}{d\phi} (e^{ikr\cos\phi\cos\theta}),$$

$$\cos\phi e^{ikr\cos\phi\cos\theta} = (-1)\frac{1}{l} \frac{d}{dr} (e^{ikr\cos\phi\cos\theta}),$$

where $l=k\cos\theta$. Using these identities the required expansions of the TM incident wave are as follows

$$(TM) \quad E_r = E_0 e^{ihz} \sum_{n=-\infty}^{\infty} i^n (i)\frac{h}{k} J'_n(lr)e^{in\phi}, \quad (76)$$

$$(TM) \quad E_\phi = E_0 e^{ihz} \sum_{n=-\infty}^{\infty} i^n (i)\frac{h}{klr}(in) J_n(lr)e^{in\phi}, \quad (77)$$

$$(TM) \quad E_z = E_0 e^{ihz} \sum_{n=-\infty}^{\infty} i^n \frac{l}{k} J_n(lr)e^{in\phi}, \quad (78)$$

$$(TM) \quad H_r = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{1}{i k r} (i n) J_n(kr) e^{in\phi}, \quad (79)$$

$$(TM) \quad H_\phi = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n (i) J_n(kr) e^{in\phi}. \quad (80)$$

Similarly, the required expansions of the TE incident wave are

$$(TE) \quad E_r = -E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{1}{i k r} (i n) J_n(kr) e^{in\phi}, \quad (81)$$

$$(TE) \quad E_\phi = -E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n (i) J_n(kr) e^{in\phi}, \quad (82)$$

$$(TE) \quad H_r = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n (i) \frac{h}{k} J'_n(kr) e^{in\phi}, \quad (83)$$

$$(TE) \quad H_\phi = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n (i) \frac{h}{k k r} (i n) J_n(kr) e^{in\phi}, \quad (84)$$

$$(TE) \quad H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{1}{k} J_n(kr) e^{in\phi}. \quad (85)$$

Having solved Maxwell's equations for the scattered fields and then obtained series expansions of the incident planewave, the total fields may be written down at once. The total electromagnetic fields in the region exterior to the cylinder, for $r > b$, are given by the sum of the incident and scattered fields. In the interior region, for $r < a$, and inside the conducting layer, $a < r < b$, the total fields are given by the expansions for the scattered fields alone. The expansion coefficients may then be determined by imposing boundary conditions on the fields at the surfaces of the conducting cylinder. This procedure shall be carried out for both the TM and TE incident waves in the following sections, sections 5 and 6. Although the full solutions shall be presented, the shielding effectiveness shall not be calculated for the case of oblique incidence due to the complexity of the solutions.

5. OBLIQUE TM INCIDENT WAVE

In this section the complete electromagnetic field solutions shall be obtained for the scattering of an obliquely incident TM planewave by a circular cylindrical conducting shield. These solutions differ from the solutions in the case of a perpendicularly incident TM wave in that the scattered fields contain both TM and TE components. Therefore, the solutions for the case of an obliquely incident planewave require eight independent sets of expansion coefficients instead of four. This is a consequence of the breaking of the perfect two-dimensional symmetry in the previous problem. For oblique incidence, it is interesting to note that if the cylinder were a perfect conductor then no mixing of the TM and TE scattered fields would occur and a TM incident wave would give rise to a TM scattered wave only, as in the case of perpendicular incidence. This implies that for a good conductor the secondary scattered modes, TE in this case, are of low intensity relative to the dominant TM mode.

Proceeding now with the solution. In the exterior region, region I, where $r > b$, the total fields are given by

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{h}{k} J'_n(lr) + \frac{h}{k} a_n H'_n(lr) + \frac{in}{lr} b_n H_n(lr) \right) e^{in\phi}, \quad (86)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{ihn}{klr} J_n(lr) + \frac{ihn}{klr} a_n H_n(lr) - b_n H'_n(lr) \right) e^{in\phi}, \quad (87)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{l}{k} J_n(lr) + \frac{l}{k} a_n H_n(lr) \right) e^{in\phi}, \quad (88)$$

$$H_r = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{in}{lr} J_n(lr) + \frac{in}{lr} a_n H_n(lr) - \frac{h}{k} b_n H'_n(lr) \right) e^{in\phi}, \quad (89)$$

$$H_\phi = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(J'_n(lr) + a_n H'_n(lr) + \frac{h}{klr} (in) b_n H_n(lr) \right) e^{in\phi}, \quad (90)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{l}{k} b_n H_n(lr) \right) e^{in\phi}. \quad (91)$$

The total fields inside the conductor, that is, in region II, where $a < r < b$, are

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{h}{k} [c_n J'_n(\chi r) + d_n H'_n(\chi r)] + \right. \\ \left. + \frac{in}{\chi r} [f_n J_n(\chi r) + g_n H_n(\chi r)] \right\} e^{in\phi}, \quad (92)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{h}{k\chi r} (in) [c_n J_n(\chi r) + d_n H_n(\chi r)] + \right. \\ \left. - [f_n J'_n(\chi r) + g_n H'_n(\chi r)] \right\} e^{in\phi}, \quad (93)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\chi}{k} [c_n J_n(\chi r) + d_n H_n(\chi r)] e^{in\phi}, \quad (94)$$

$$H_r = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \left(\frac{\sigma - i\omega\epsilon}{i\omega\epsilon_0} \right) \frac{in}{\chi r} [c_n J_n(\chi r) + d_n H_n(\chi r)] + \right. \\ \left. + \frac{h}{k} [f_n J'_n(\chi r) + g_n H'_n(\chi r)] \right\} e^{in\phi}, \quad (95)$$

$$H_\phi = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \left(\frac{\sigma - i\omega\epsilon}{i\omega\epsilon_0} \right) [c_n J'_n(\chi r) + d_n H'_n(\chi r)] + \right. \\ \left. - \frac{h}{k\chi r} (in) [f_n J_n(\chi r) + g_n H_n(\chi r)] \right\} e^{in\phi}, \quad (96)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\chi}{k} [f_n J_n(\chi r) + g_n H_n(\chi r)] e^{in\phi}. \quad (97)$$

Note that the term $(\sigma - i\omega\epsilon)/(i\omega\epsilon_0)$ in equations 95 and 96 is equal to $-\gamma^2/k^2$.

The total fields on the interior of the shield, in region III, where $r < a$, are given by

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{h}{k} p_n J'_n(\ell r) + \frac{in}{\ell r} q_n J_n(\ell r) \right) e^{in\phi}, \quad (98)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{h}{k\ell r} (in) p_n J_n(\ell r) - q_n J'_n(\ell r) \right) e^{in\phi}, \quad (99)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\ell}{k} p_n J_n(\ell r) e^{in\phi}, \quad (100)$$

$$H_r = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{in}{\ell r} p_n J_n(\ell r) - \frac{h}{k} q_n J'_n(\ell r) \right) e^{in\phi}, \quad (101)$$

$$H_\phi = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(p_n J'_n(\ell r) + \frac{h}{k\ell r} (in) q_n J_n(\ell r) \right) e^{in\phi}, \quad (102)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\ell}{k} q_n J_n(\ell r) e^{in\phi}. \quad (103)$$

To determine the expansion coefficients the boundary conditions must be applied at the surfaces $r=a$ and $r=b$. The boundary conditions require that the tangential components of the E-field and the H-field be continuous on the boundary surfaces. The condition that E_ϕ is continuous at $r=b$ implies

$$\begin{aligned} & \frac{h}{k\ell b} (in) J_n(\ell b) + \frac{h}{k\ell b} (in) a_n H_n(\ell b) - b_n H'_n(\ell b) = \\ & = \frac{h}{k\chi b} (in) [c_n J_n(\chi b) + d_n H_n(\chi b)] - [f_n J'_n(\chi b) + g_n H'_n(\chi b)]. \end{aligned} \quad (104)$$

The condition that E_z is continuous at $r=b$ implies

$$\frac{\ell}{k} J_n(\ell b) + \frac{\ell}{k} a_n H_n(\ell b) = \frac{\chi}{k} [c_n J_n(\chi b) + d_n H_n(\chi b)]. \quad (105)$$

The requirement that H_ϕ is continuous at $r=b$ implies

$$J'_n(\ell b) + a_n H'_n(\ell b) + \frac{h}{k\ell b}(in)b_n H_n(\ell b) = \frac{\chi^2}{k} [c_n J'_n(\chi b) + d_n H'_n(\chi b)] + \frac{h}{k\chi b}(in)[f_n J_n(\chi b) + g_n H_n(\chi b)]. \quad (106)$$

The requirement that H_z is continuous at $r=b$ implies

$$\frac{\ell}{k} b_n H_n(\ell b) = \frac{\chi}{k} [f_n J_n(\chi b) + g_n H_n(\chi b)]. \quad (107)$$

The requirement that E_ϕ is continuous at $r=a$ implies

$$\begin{aligned} \frac{h}{k\chi a}(in)[c_n J_n(\chi a) + d_n H_n(\chi a)] - [f_n J'_n(\chi a) + g_n H'_n(\chi a)] &= \\ = \frac{h}{k\ell a}(in) p_n J_n(\ell a) - q_n J'_n(\ell a). \end{aligned} \quad (108)$$

The requirement that E_z is continuous at $r=a$ implies

$$\frac{\chi}{k} [c_n J_n(\chi a) + d_n H_n(\chi a)] = \frac{\ell}{k} p_n J_n(\ell a). \quad (109)$$

The requirement that H_ϕ is continuous at $r=a$ implies

$$\begin{aligned} \frac{\gamma^2}{k^2} [c_n J'_n(\chi a) + d_n H'_n(\chi a)] + \frac{h}{k \chi a} (in) [f_n J_n(\chi a) + g_n H_n(\chi a)] = \\ = p_n J'_n(\ell a) + \frac{h}{k \ell a} (in) q_n J_n(\ell a). \end{aligned} \quad (110)$$

And, the requirement that H_z is continuous at $r=a$ implies

$$\frac{\chi}{k} [f_n J_n(\chi a) + g_n H_n(\chi a)] = \frac{\ell}{k} q_n J_n(\ell a). \quad (111)$$

These are all of the required boundary conditions. The full set of boundary conditions constitutes a nonhomogeneous set of eight equations in the eight unknowns a_n , b_n , c_n , d_n , f_n , g_n , p_n , and q_n . This system may be solved by using the standard technique. The explicit solutions for the coefficients is straightforward and shall not be given here. According to Kramer's rule, the solutions are given by relations of the form

$$\begin{aligned} a_n = \frac{U_1}{V}, \quad b_n = \frac{U_2}{V}, \quad c_n = \frac{U_3}{V}, \quad d_n = \frac{U_4}{V}, \\ f_n = \frac{U_5}{V}, \quad g_n = \frac{U_6}{V}, \quad p_n = \frac{U_7}{V}, \quad q_n = \frac{U_8}{V}; \end{aligned} \quad (112)$$

where the determinants U_i and V are determined by inspection of the eight equations 104, 105, 106, 107, 108, 109, 110, and 111.

This completes the solutions for the electromagnetic fields for the scattering of an obliquely incident TM planewave. Due to the fact that the expansion coefficients are given by eight-by-eight determinants the shielding effectiveness for the case of oblique incidence shall not be calculated.

6. OBLIQUE TE INCIDENT WAVE

In this section the complete electromagnetic field solutions shall be obtained for the scattering of an obliquely incident TE planewave by a circular cylindrical conducting shield. As was noted in the last section these solutions differ from the solutions in the case of a perpendicularly incident TM wave in that the scattered fields contain both TM and TE components. However, if the cylinder were a perfect conductor then no mixing of the TM and TE scattered fields would occur and a TE incident wave would give rise to a TE scattered wave only, as in the case of perpendicular incidence. This implies that for a good conductor the secondary scattered modes, TM in this case, are of low intensity relative to the dominant TE mode.

The expressions for the scattered fields in each of the three regions are the same as in section 5 with the expansion coefficients replaced with capital letters: A, B, C, D, F, G, P, and Q. Only the incident fields are different. The development of the solutions is identical to that in section 5. Proceeding now with the solution, in the exterior region, region I, where $r > b$, the total fields are given by

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{in}{kr} J_n(kr) + \frac{h}{k} A_n H'_n(kr) + \frac{in}{kr} b_n H_n(kr) \right) e^{in\phi}, \quad (113)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(-J_n(kr) + \frac{ihn}{kkr} A_n H_n(kr) - b_n H'_n(kr) \right) e^{in\phi}, \quad (114)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{k}{k} A_n H_n(kr) e^{in\phi}, \quad (115)$$

$$H_r = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(-\frac{h}{k} J'_n(kr) + \frac{in}{kr} A_n H_n(kr) - \frac{h}{k} B_n H'_n(kr) \right) e^{in\phi}, \quad (116)$$

$$H_\phi = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{ihn}{kkr} J_n(kr) + A_n H'_n(kr) + \frac{ihn}{kkr} B_n H_n(kr) \right) e^{in\phi}, \quad (117)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{k}{k} J_n(kr) + \frac{k}{k} B_n H_n(kr) \right) e^{in\phi}. \quad (118)$$

The total fields inside the conductor, that is, in region II, where $a < r < b$, are

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{h}{k} [C_n J'_n(\chi r) + D_n H'_n(\chi r)] + \right. \\ \left. + \frac{in}{\chi r} [F_n J_n(\chi r) + G_n H_n(\chi r)] \right\} e^{in\phi}, \quad (119)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{h}{k\chi r} (in) [C_n J_n(\chi r) + D_n H_n(\chi r)] + \right. \\ \left. - [F_n J'_n(\chi r) + G_n H'_n(\chi r)] \right\} e^{in\phi}, \quad (120)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\chi}{k} [C_n J_n(\chi r) + D_n H_n(\chi r)] e^{in\phi}, \quad (121)$$

$$H_r = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{in}{\chi r} \frac{\chi^2}{k^2} [C_n J_n(\chi r) + D_n H_n(\chi r)] + \right. \\ \left. - \frac{h}{k} [F_n J'_n(\chi r) + G_n H'_n(\chi r)] \right\} e^{in\phi}, \quad (122)$$

$$H_\phi = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left\{ \frac{\chi^2}{k^2} [C_n J'_n(\chi r) + D_n H'_n(\chi r)] + \right. \\ \left. + \frac{h}{k\chi r} (in) [F_n J_n(\chi r) + G_n H_n(\chi r)] \right\} e^{in\phi}, \quad (123)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\chi}{k} [F_n J_n(\chi r) + G_n H_n(\chi r)] e^{in\phi}. \quad (124)$$

The total fields on the interior of the shield, in region III, where $r < a$, are given by

$$E_r = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{h}{k} P_n J'_n(\ell r) + \frac{in}{\ell r} Q_n J_n(\ell r) \right) e^{in\phi}, \quad (125)$$

$$E_\phi = iE_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{h}{k\ell r} (in) P_n J_n(\ell r) - Q_n J'_n(\ell r) \right) e^{in\phi}, \quad (126)$$

$$E_z = E_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\ell}{k} P_n J_n(\ell r) e^{in\phi}, \quad (127)$$

$$H_r = -iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(\frac{in}{\ell r} P_n J_n(\ell r) - \frac{h}{k} Q_n J'_n(\ell r) \right) e^{in\phi}, \quad (128)$$

$$H_\phi = iH_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \left(P_n J'_n(\ell r) + \frac{h}{k\ell r} (in) Q_n J_n(\ell r) \right) e^{in\phi}, \quad (129)$$

$$H_z = H_0 e^{ihz} \sum_{-\infty}^{\infty} i^n \frac{\ell}{k} Q_n J_n(\ell r) e^{in\phi}. \quad (130)$$

To determine the expansion coefficients the boundary conditions must be applied at the surfaces $r=a$ and $r=b$. The boundary conditions require that the tangential components of the E- and the H-fields be continuous on the boundary surfaces. The condition that E_ϕ is continuous at $r=b$ implies

$$\begin{aligned} & -J_n(\ell b) + \frac{h}{k\ell b} (in) A_n H_n(\ell b) - B_n H'_n(\ell b) = \\ & = \frac{h}{k\chi b} (in) [C_n J_n(\chi b) + D_n H_n(\chi b)] - [F_n J'_n(\chi b) + G_n H'_n(\chi b)]. \end{aligned} \quad (131)$$

The condition that E_z is continuous at $r=b$ implies

$$\frac{\ell}{k} A_n H_n(\ell b) = \frac{\chi}{k} [C_n J_n(\chi b) + D_n H_n(\chi b)]. \quad (132)$$

The requirement that H_ϕ is continuous at $r=b$ implies

$$\begin{aligned} \frac{h}{k\ell b} (in) J_n(\ell b) + A_n H'_n(\ell b) + \frac{h}{k\ell b} (in) B_n H_n(\ell b) = \\ \frac{\chi^2}{k^2} [C_n J'_n(\chi b) + D_n H'_n(\chi b)] + \frac{h}{k\chi b} (in) [F_n J_n(\chi b) + G_n H_n(\chi b)]. \end{aligned} \quad (133)$$

The requirement that H_z is continuous at $r=b$ implies

$$\frac{\ell}{k} J_n(\ell b) + \frac{\ell}{k} B_n H_n(\ell b) = \frac{\chi}{k} [F_n J_n(\chi b) + G_n H_n(\chi b)]. \quad (134)$$

The requirement that E_ϕ is continuous at $r=a$ implies

$$\begin{aligned} \frac{h}{k\chi a} (in) [C_n J_n(\chi a) + D_n H_n(\chi a)] - [F_n J'_n(\chi a) + G_n H'_n(\chi a)] = \\ = \frac{h}{k\ell a} (in) P_n J_n(\ell a) - Q_n J'_n(\ell a). \end{aligned} \quad (135)$$

The requirement that E_z is continuous at $r=a$ implies

$$\frac{\chi}{k} [C_n J_n(\chi a) + D_n H_n(\chi a)] = \frac{\ell}{k} P_n J_n(\ell a). \quad (136)$$

The requirement that H_ϕ is continuous at $r=a$ implies

$$\begin{aligned}
\frac{\gamma^2}{k^2} [C_n J'_n(\chi a) + D_n H'_n(\chi a)] + \frac{h}{k \chi a} (\ln) [F_n J_n(\chi a) + G_n H_n(\chi a)] = \\
= P_n J'_n(\ell a) + \frac{h}{k \ell a} (\ln) Q_n J_n(\ell a).
\end{aligned} \quad (137)$$

And, the requirement that H_z is continuous at $r=a$ implies

$$\frac{\chi}{k} [F_n J_n(\chi a) + G_n H_n(\chi a)] = \frac{\ell}{k} Q_n J_n(\ell a). \quad (138)$$

These are all of the required boundary conditions. The full set of boundary conditions constitutes a nonhomogeneous set of eight equations in the eight unknowns A_n , B_n , C_n , D_n , F_n , G_n , P_n , and Q_n . This system may be solved by the standard procedure. The explicit solutions for the coefficients is straightforward and shall not be given here. According to Kramer's rule, the solutions are given by relations of the form

$$\begin{aligned}
A_n = \frac{U_1}{V}, \quad B_n = \frac{U_2}{V}, \quad C_n = \frac{U_3}{V}, \quad D_n = \frac{U_4}{V}, \\
F_n = \frac{U_5}{V}, \quad G_n = \frac{U_6}{V}, \quad P_n = \frac{U_7}{V}, \quad Q_n = \frac{U_8}{V};
\end{aligned} \quad (139)$$

where the determinants U_i and V are determined by inspection of the eight equations 131, 132, 133, 134, 135, 136, 137, and 138. This completes the solutions for the electromagnetic fields for the case of oblique incidence. Due to the fact that the expansion coefficients are given by eight-by-eight determinants the shielding effectiveness for the case of oblique incidence shall not be calculated.

Although the solutions of the fields is somewhat more complicated than in the case of perpendicular incidence, the fact that for a good conductor only a single mode is dominant, implies that the behavior of the shielding effectiveness should be very similar to the case of perpendicular incidence. While this is almost certainly true for near perpendicular incidence, for large angles of incidence, and especially grazing incidence, this can only be verified through calculation.

7. SHIELDING EFFECTIVENESS OF THE CYLINDER

The main purpose of this investigation is to calculate and plot the shielding effectiveness verses frequency for hollow cylindrical conducting shields. Results shall be presented only for the case of perpendicular incidence. For perpendicular incidence, the shielding effectiveness is defined for the TM incident wave, the E-polarized wave, as the ratio of the magnitude squared of the E-field on the axis of the cylinder to the magnitude squared of the E-field of the incident wave

$$\text{(TM)} \quad SE_e = \left| \frac{E_{\text{axis}}}{E_{\text{inc}}} \right|^2. \quad (140)$$

Similarly, the shielding effectiveness for the TE incident wave is defined as the ratio of the magnitude squared of the H-field on the axis to the magnitude of the H-field of the incident wave

$$\text{(TE)} \quad SE_h = \left| \frac{H_{\text{axis}}}{H_{\text{inc}}} \right|^2. \quad (141)$$

The shielding effectiveness is usually expressed in terms of dB notation. For equations 140 and 141 the shielding effectiveness in dB is

$$\text{(TM)} \quad SE_e(\text{dB}) = -20 \log \left| \frac{E_{\text{axis}}}{E_{\text{inc}}} \right|. \quad (142)$$

$$\text{(TE)} \quad SE_h(\text{dB}) = -20 \log \left| \frac{H_{\text{axis}}}{H_{\text{inc}}} \right|. \quad (143)$$

To calculate the shielding effectiveness the values of the fields on the axis must be determined. Using the series expansion of the E-field in region III for the TM incident wave, equation 29,

together with the fact that $J_n(0) = 1$ if $n=0$, and $J_n(0) = 0$ if $n=\pm 1, \pm 2, \dots$, the field on axis is given by

$$(TM) \quad E_{axis} = E_0 d_0. \quad (144)$$

Similarly, from the series expansion of the H-field for the TE case, equation 47, the field on axis is given by

$$(TE) \quad H_{axis} = H_0 D_0. \quad (145)$$

Consequently, the shielding effectiveness for the TM and TE incident waves are

$$(TM) \quad SE_e = |d_0|^2, \quad (146)$$

$$(TE) \quad SE_h = |D_0|^2. \quad (147)$$

The quantities d_0 and D_0 are evaluated in Appendix B and Appendix C, respectively. For the case of perpendicular incidence the final results for the shielding effectiveness for the TM and TE incident waves take the following forms

$$(TM) \quad SE_e^{-1/2} = |\lambda\alpha + \mu\beta| \frac{4}{\pi^2 ab}, \quad (148)$$

$$(TE) \quad SE_h^{-1/2} = |\lambda\alpha' + \mu\beta'| \frac{4}{\pi^2 ab}, \quad (149)$$

where

$$\lambda = 2Ae^{i\frac{\pi}{4}} \sin(\delta + i\delta), \quad (150)$$

$$\mu = 2Ae^{i\frac{\pi}{4}} \cos(\delta + i\delta), \quad (151)$$

$$\alpha = \gamma^2 H_0(kb) J_0(ka) + k^2 H_1(kb) J_1(ka), \quad (152)$$

$$\beta = \gamma k [J_0(ka) H_1(kb) - H_0(kb) J_1(ka)], \quad (153)$$

$$\alpha' = k^2 H_0(kb) J_0(ka) + \gamma^2 H_1(kb) J_1(ka), \quad (154)$$

and

$$\beta' = \gamma k [J_0(ka) H_1(kb) - J_1(ka) H_0(kb)], \quad (155)$$

where

$$A = \frac{1}{\pi} |\gamma a|^{-1/2} |\gamma b|^{-1/2}, \quad (156)$$

and

$$\delta = 2^{-1/2} |\gamma| (b-a). \quad (157)$$

These results were derived by approximating the Bessel and Hankel functions having the complex arguments $z=\gamma a$ and $z=\gamma b$, by their principle asymptotic forms. Consequently, these equations are not exact, and they do not hold for all frequencies. However, as discussed in Appendix B, these equations are sufficiently accurate for all frequencies in the range $\sqrt{\omega\mu_0}a > 125$.

The shielding effectiveness was calculated for a conducting cylinder having an outer diameter of 155 mm., conductivity $10^7 \text{ ohm}^{-1}\text{m}^{-1}$, and thickness 0.1 mm. The results for the TM case are shown as a solid curve in figure 3, and the results for the TE case are shown as the solid curve in figure 4. The low frequency behavior is not very interesting; below 10^6 Hz. the shielding effectiveness decreases to 0 dB almost as a straight line on a log-log plot like figure 3. For higher frequencies the shielding effectiveness increases rapidly as is expected by comparison with the behavior of a planar shield. Furthermore a fine structure appears. It is well known that a pair of parallel conducting planes gives rise to Fabry-Perot resonances, causing increased transmission of the incident beam, when the distance between the plates is equal to integer multiples of a half wavelength. A similar resonant behavior is expected for the cylindrical conducting shell, causing an increase in the penetration of the cylinder by the incident wave. This is born out by the theory. As can be seen in figures 3 and 4, the shielding effectiveness shows sharp dips at the resonant frequencies given by the zeros of the equations

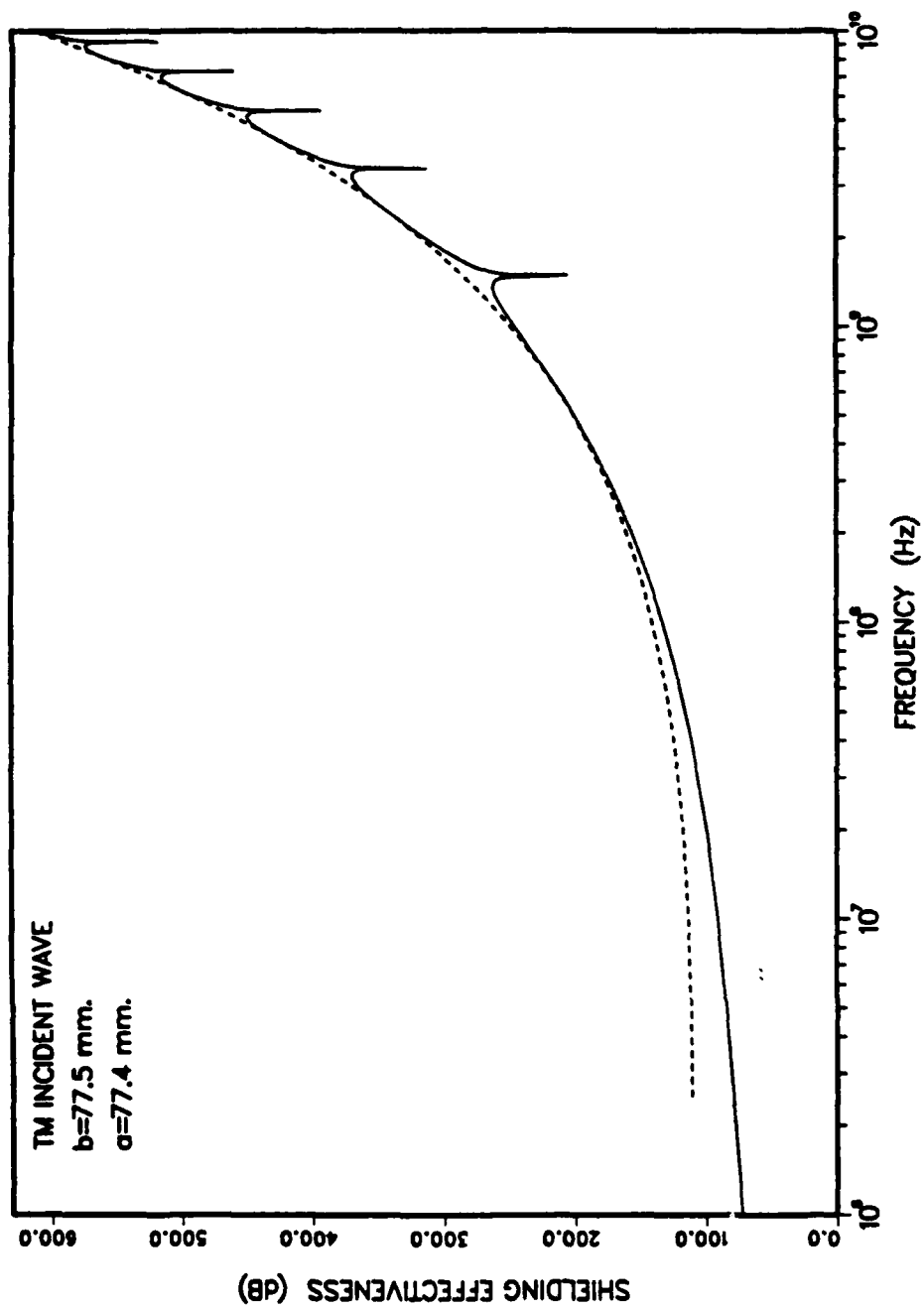


Figure 3. Shielding effectiveness for the cylinder: TM incident wave

$$J_0(ka)=0, \quad \text{for the TM incident wave, and} \quad (158)$$

$$J_1(ka)=0, \quad \text{for the TE incident wave.} \quad (159)$$

A derivation of the resonance conditions, equations 158 and 159, is given in Appendixes B and C. The shielding effectiveness drops by about 3 or 4 orders of magnitude at the resonance frequencies in this example. Nevertheless, in practice, and especially for thick shields this effect is negligible. Calculations of the shielding effectiveness for a thick shield having a 1.0 mm thickness prove this. The resonances are much less pronounced for the thicker shield due to the overall magnitude of the attenuation of the shield.

Using the solutions for the shielding effectiveness developed in Appendix B and Appendix C an approximate closed form result for the shielding effectiveness is derived in Appendix D. The approximate formula, which is identical for the TM and TE modes, is given by

$$SE = \frac{4\omega\epsilon_0}{\sigma} e^{-2\delta}, \quad (160)$$

or, in terms of dB,

$$SE_{dB} = 10\left\{\log\left(\frac{\sigma}{4\epsilon_0}\right) - \log(\omega) + (2\sigma\mu_0)^{1/2}(b-a)\log(e) \omega^{1/2}\right\}. \quad (161)$$

The shielding effectiveness calculated by using equation 161 is plotted as a dashed curve in figures 3 and 4. The agreement with the exact solutions is very good for frequencies in the resonance regime. For lower frequencies this approximation breaks down. As is shown in Appendix D, this formula diverges as ω approaches zero and therefore cannot be used for frequencies below

$$\omega_0 = \frac{2}{\mu_0\sigma} \frac{1}{(b-a)^2}. \quad (162)$$

However, the approximate formula 161 is very useful since it allows shielding calculations to be made quickly on a hand calculator without the need for complicated mathematics or computer programs.

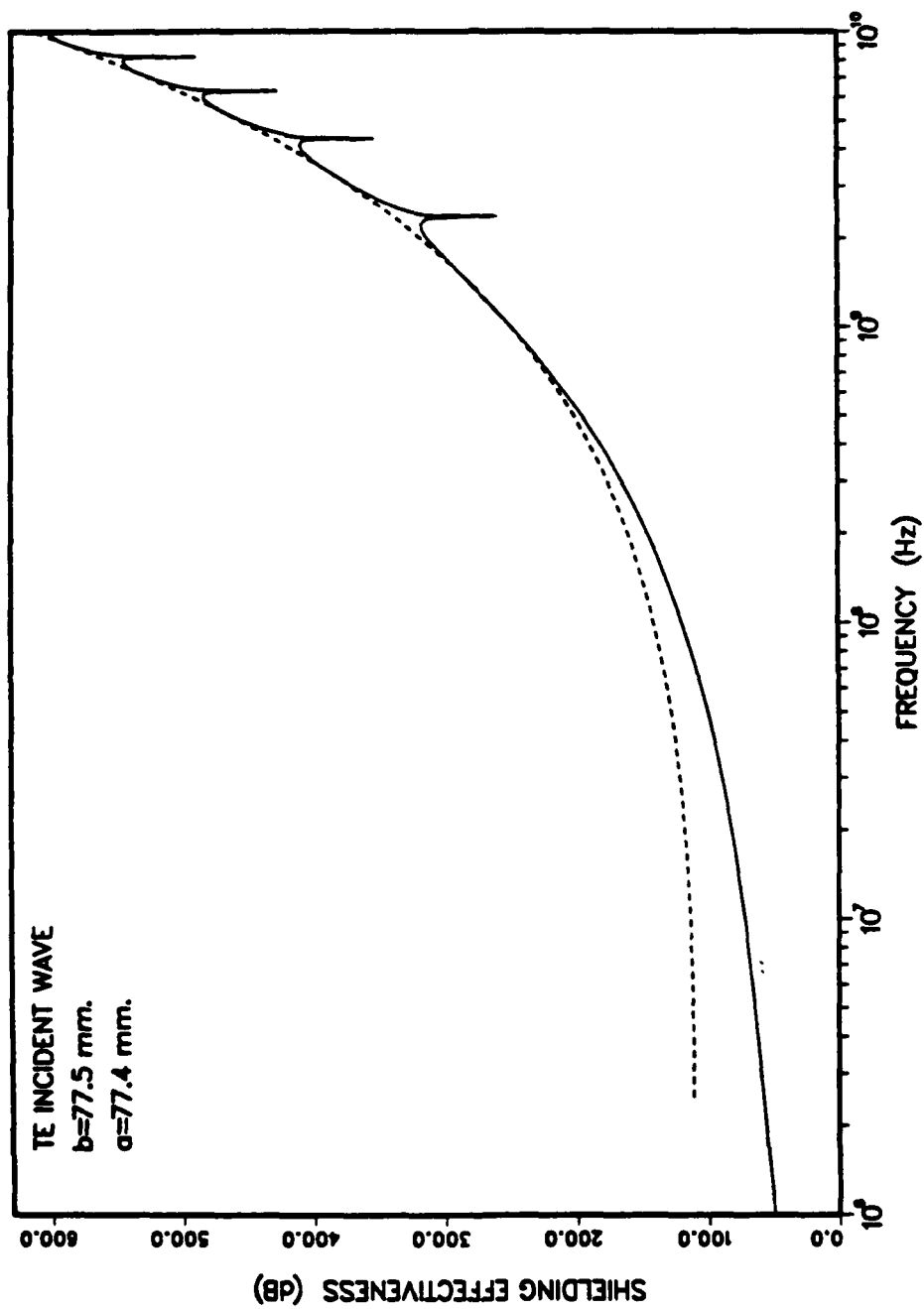


Figure 4. Shielding effectiveness for the cylinder: TE incident wave

8. ELECTROMAGNETIC FIELDS FOR THE SPHERE

A conducting sphere with inner radius $r=a$ and outer radius $r=b$ is situated in empty space. The material parameters of the conductor are ϵ , μ_0 , and σ . The conductor is assumed to be a non-magnetic material, so it is possible to set $\mu=\mu_0$. A linearly polarized planewave of frequency f is incident on the sphere. The planewave propagates along the z -direction of a cartesian coordinate system with the origin at the center of the sphere. The total field in the region exterior to the sphere is given by the sum of the incident field and the scattered field, that is,

$$\mathbf{E} = \mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{scat}}.$$

The time dependence of the field vectors is the same as that of the incident wave, which is assumed to be $e^{-i\omega t}$, where the imaginary unit $i^2 = -1$ is used to prevent confusion with the spherical Bessel function $j(z)$. The fields of the incident planewave are

$$E_x = E_0 e^{ikz}, \quad (163)$$

$$H_y = H_0 e^{ikz}, \quad (164)$$

where $k=\omega/c$, $c=1/\sqrt{\mu_0\epsilon_0}$ is the speed of light, and $H_0=\sqrt{\epsilon_0/\mu_0} E_0$. Following Stratton, the fields may be expanded in terms of the vector spherical wavefunctions L , M , and N , since these functions form a complete, orthogonal set of solutions to the vector wave equation in spherical coordinates. The procedure is well known. The fields of the incident planewave may be expressed by the expansions

$$\mathbf{E}_{\text{inc}} = E_0 e^{ikz} \mathbf{e}_x = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [M_{0,1n}^1 - iN_{e,1n}^1]_{\rho=kr}, \quad (165)$$

$$\mathbf{H}_{\text{inc}} = H_0 e^{ikz} \mathbf{e}_y = -H_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [M_{e,1n}^1 + iN_{0,1n}^1]_{\rho=kr}, \quad (166)$$

where the vector spherical wavefunctions are given by

$$M_{O,1n}^1 = \frac{+1}{\sin\theta} j_n(\rho) P_n^1(\cos\theta) \cos\phi e_\theta - j_n(\rho) \frac{dP_n^1}{d\theta} \sin\phi e_\phi, \quad (167)$$

$$M_{E,1n}^1 = \frac{-1}{\sin\theta} j_n(\rho) P_n^1(\cos\theta) \sin\phi e_\theta - j_n(\rho) \frac{dP_n^1}{d\theta} \cos\phi e_\phi, \quad (168)$$

and

$$\begin{aligned} N_{O,1n}^1 = & \frac{n(n+1)}{\rho} j_n(\rho) P_n^1(\cos\theta) \sin\phi e_r + \frac{1}{\rho} [\rho j_n(\rho)]' \frac{dP_n^1}{d\theta} \sin\phi e_\theta + \\ & + \frac{1}{\rho \sin\theta} [\rho j_n(\rho)]' P_n^1(\cos\theta) \cos\phi e_\phi, \end{aligned} \quad (169)$$

$$\begin{aligned} N_{E,1n}^1 = & \frac{n(n+1)}{\rho} j_n(\rho) P_n^1(\cos\theta) \cos\phi e_r + \frac{1}{\rho} [\rho j_n(\rho)]' \frac{dP_n^1}{d\theta} \cos\phi e_\theta + \\ & - \frac{1}{\rho \sin\theta} [\rho j_n(\rho)]' P_n^1(\cos\theta) \sin\phi e_\phi, \end{aligned} \quad (170)$$

the prime denoting differentiation with respect to ρ . The function $j_n(\rho)$ is the spherical Bessel function, and the function $P_n^1(\cos\theta)$ is the associated Legendre polynomial with upper index $m=1$. The wavefunction L is not required for expansions of the E and H fields since the divergence of L is not zero. The wavefunctions L , M , and N compose a fundamental set of solutions of the vector Helmholtz equation, in spherical coordinates

$$-\nabla \times \nabla \times F + k^2 F = 0. \quad (171)$$

Furthermore, the functions M and N satisfy the relations

$$\nabla \times M = kN, \quad (172)$$

$$\nabla \times N = kM. \quad (173)$$

A derivation of the vector spherical wave functions, their orthogonality relations, and other properties is given in [xx]. The scattered fields outside the sphere may be expressed in terms of series expansions analogous to those used for the incident fields. For $r > b$ the scattered fields are given by

$$E_{\text{scat}} = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [a_n^{M^3, 1n} - i b_n^{N^3, 1n}] \rho = kr, \quad (174)$$

$$H_{\text{scat}} = -H_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [b_n^{M^3, 1n} + i a_n^{N^3, 1n}] \rho = kr, \quad (175)$$

where the wavefunctions with the superscript 3, M^3 and N^3 , are obtained from the defining relations for M^1 and N^1 by replacing the spherical Bessel function $j_n(\rho)$ with the spherical Hankle function of the first kind $h_n^1(\rho)$. Next, similar expansions for the fields inside the conducting shell shall be obtained.

Assuming a time dependence of the form $e^{-i\omega t}$, Maxwell's equations inside the conductor are given by

$$\nabla \times E = i\omega \mu_0 H, \quad (176)$$

$$\nabla \times H = (\sigma - i\omega \epsilon) E, \quad (177)$$

$$\nabla \cdot E = 0, \quad (178)$$

$$\nabla \cdot H = 0, \quad (179)$$

where the material equations are

$$D = \epsilon E, \quad (180)$$

$$B = \mu_0 H, \quad (181)$$

$$J = \sigma E. \quad (182)$$

The field equations imply that the field vectors E and H satisfy the vector wave equation

$$\nabla \times \nabla \times \mathbf{E} + \gamma^2 \mathbf{E} = 0, \quad (183)$$

$$\nabla \times \nabla \times \mathbf{H} + \gamma^2 \mathbf{H} = 0, \quad (184)$$

where

$$\gamma^2 = i\omega\mu_0(\sigma - i\omega\epsilon). \quad (185)$$

Therefore, the fields inside the conductor, for $a < r < b$, have the expansions

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [c_n M_{0,1n}^s + d_n M_{0,1n}^i - i f_n N_{e,1n}^s - i g_n N_{e,1n}^i]_{\rho=yr}, \quad (186)$$

$$\mathbf{H} = -\frac{\gamma}{k} \mathbf{H}_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [f_n M_{e,1n}^s + g_n M_{e,1n}^i + i c_n N_{0,1n}^s + i d_n N_{0,1n}^i]_{\rho=yr}. \quad (187)$$

Wavefunctions of both the first and third kinds are required to be able to match the boundary conditions with both the exterior and interior fields. Similarly, the fields inside the shielded volume, $r < a$, have the following expansions

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [p_n M_{0,1n}^i - i q_n N_{e,1n}^i]_{\rho=kr}, \quad (188)$$

$$\mathbf{H} = -\mathbf{H}_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} [q_n M_{e,1n}^i + i p_n N_{0,1n}^i]_{\rho=kr}. \quad (189)$$

Only the wavefunctions of the first kind may be used in these expansions since the fields must be finite at the origin.

The next step in the solution of the boundary value problem is to match boundary conditions. The boundary conditions on the field vectors require that the tangential components of the field vectors \mathbf{E} and \mathbf{H} must be continuous at the boundaries $r=a$ and $r=b$. The condition that the tangential components of \mathbf{E} are continuous at $r=b$ implies

$$j_n(kb) + a_n h_n(kb) = c_n h_n(\gamma b) + d_n j_n(\gamma b), \quad (190)$$

$$\frac{\gamma}{k} \{ [\rho j_n(\rho)]' + b_n [\rho h_n(\rho)]' \}_{\rho=kb} = \{ f_n [\rho h_n(\rho)]' + g_n [\rho j_n(\rho)]' \}_{\rho=\gamma b}. \quad (191)$$

The condition that the tangential components of H are continuous at $r=b$ implies

$$j_n(kb) + b_n h_n(kb) = \frac{\gamma}{k} [f_n h_n(\gamma b) + g_n j_n(\gamma b)], \quad (192)$$

$$\{ [\rho j_n(\rho)]' + a_n [\rho h_n(\rho)]' \}_{\rho=kb} = \{ c_n [\rho h_n(\rho)]' + d_n [\rho j_n(\rho)]' \}_{\rho=\gamma b}. \quad (193)$$

The condition that the tangential components of E are continuous at $r=a$ implies

$$c_n h_n(\gamma a) + d_n j_n(\gamma a) = p_n j_n(ka), \quad (194)$$

$$\{ f_n [\rho h_n(\rho)]' + g_n [\rho j_n(\rho)]' \}_{\rho=\gamma a} = \frac{\gamma}{k} q_n \{ [\rho j_n(\rho)]' \}_{\rho=ka}. \quad (195)$$

And lastly, the condition that the tangential components of H are continuous at $r=a$ implies

$$\frac{\gamma}{k} [f_n h_n(\gamma a) + g_n j_n(\gamma a)] = q_n j_n(ka), \quad (196)$$

$$\{ c_n [\rho h_n(\rho)]' + d_n [\rho j_n(\rho)]' \}_{\rho=\gamma a} = p_n \{ [\rho j_n(\rho)]' \}_{\rho=ka}. \quad (197)$$

These are all the required boundary conditions. This set of equations constitute a nonhomogeneous system of eight equations in eight unknowns. The solutions of this system yield the expansion coefficients for the fields. To investigate the shielding properties of the conducting sphere it is necessary to solve for the fields inside the shielded volume. This implies that it is necessary to solve for the coefficients p_n and q_n . To avoid the direct evaluation of 8×8 determinants the system shall

be reduced to a system of 4×4 equations which shall then be solved by Kramer's rule. The coefficients a_n , b_n , p_n , and q_n shall first be eliminated from the eight equations to yield four equations in the four unknowns c_n , d_n , f_n , and g_n . Having solved the 4×4 system of equations, the coefficient p_n may be calculated from c_n and d_n by using equation 194, and the coefficient q_n may be calculated from f_n and g_n by using equation 196. To eliminate a_n , solve equation 190 for a_n , and then substitute the result into equation 193. Omitting the subscript n from all the expansion coefficients and all the Bessel and Hankel functions for simplicity, this procedure yields the result

$$\begin{aligned} c[\Gamma_h(\gamma b)h(kb) - \Gamma_h(kb)h(\gamma b)] + d[\Gamma_j(\gamma b)h(kb) - \Gamma_h(kb)j(\gamma b)] &= \\ &= \Gamma_j(kb)h(kb) - \Gamma_h(kb)j(kb), \end{aligned} \quad (198)$$

where

$$\begin{aligned} \Gamma_h(\gamma b) &= \{[\rho h_n(\rho)]'\}_{\rho=\gamma b}, \\ \Gamma_j(kb) &= \{[\rho j_n(\rho)]'\}_{\rho=kb}, \text{ etc.} \end{aligned} \quad (199)$$

The right-hand-side of equation 198 may be simplified further by taking the derivative in the Γ 's yielding

$$\rho[j'(\rho)h(\rho) - h'(\rho)j(\rho)]_{\rho=kb};$$

then, substituting $h(\rho) = j(\rho) + iy(\rho)$, this equals

$$-i\rho[j_n(\rho)y'_n(\rho) - h'_n(\rho)y_n(\rho)]_{\rho=kb}.$$

The bracketed term is the Wronskian of $j_n(\rho)$ and $y_n(\rho)$, which equals $1/\rho^2$. Thus, the left-hand side of equation 198 takes the form

$$\Gamma_j(kb)h(kb) - \Gamma_h(kb)j(kb) = \frac{-i}{kb}.$$

To recapitulate, equation 198 takes the simplified form

$$c[\Gamma_h(\gamma b)h(kb) - \Gamma_h(kb)h(\gamma b)] + d[\Gamma_j(\gamma b)h(kb) - \Gamma_h(kb)j(\gamma b)] = \frac{-1}{k\bar{b}}. \quad (200)$$

Next, to eliminate b_n solve equation 192 for b_n , and then substitute the result into equation 191. This yields

$$\begin{aligned} f[\Gamma_h(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)h(\gamma b)] + g[\Gamma_j(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)j(\gamma b)] &= \\ &= \frac{\gamma}{k} [\Gamma_j(kb)h(kb) - \Gamma_h(kb)j(kb)] \\ &= \frac{\gamma}{k} \rho[j'_n(\rho)h_n(\rho) - h'_n(\rho)j_n(\rho)]_{\rho=kb} \\ &= \frac{\gamma}{k} \frac{-1}{k\bar{b}}. \end{aligned} \quad (201)$$

To eliminate p_n , solve equation 194 for p_n , and substitute the result into equation 197. This yields the equation

$$c[\Gamma_h(\gamma a)j(ka) - \Gamma_j(ka)h(\gamma a)] + d[\Gamma_j(\gamma a)j(ka) - \Gamma_j(ka)j(\gamma a)] = 0. \quad (202)$$

Lastly, to eliminate q_n solve equation 196 for q_n and substitute the result into equation 195. This yields

$$\begin{aligned} f[\Gamma_h(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)h(\gamma a)] + \\ + g[\Gamma_j(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)j(\gamma a)] = 0. \end{aligned} \quad (203)$$

The reduced system of equations is given by the four equations

$$c[\Gamma_h(\gamma b)h(kb) - \Gamma_h(kb)h(\gamma b)] + d[\Gamma_j(\gamma b)h(kb) - \Gamma_h(kb)j(\gamma b)] = \frac{-1}{kb}$$

$$c[\Gamma_h(\gamma a)j(ka) - \Gamma_j(ka)h(\gamma a)] + d[\Gamma_j(\gamma a)j(ka) - \Gamma_j(ka)j(\gamma a)] = 0$$

$$f[\Gamma_h(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)h(\gamma b)] + g[\Gamma_j(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)j(\gamma b)] = \frac{\gamma}{k} \frac{-1}{kb}$$

$$f[\Gamma_h(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)h(\gamma a)] + g[\Gamma_j(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)j(\gamma a)] = 0.$$

The matrix of coefficients has the convenient block form

$$D = \begin{vmatrix} W_1 & 0 & 0 \\ 0 & 0 & W_2 \\ 0 & 0 & \end{vmatrix}, \quad (204)$$

where $W_1 =$

$$\begin{vmatrix} [\Gamma_h(\gamma b)h(kb) - \Gamma_h(kb)h(\gamma b)] & [\Gamma_j(\gamma b)h(kb) - \Gamma_h(kb)j(\gamma b)] \\ [\Gamma_h(\gamma a)j(ka) - \Gamma_j(ka)h(\gamma a)] & [\Gamma_j(\gamma a)j(ka) - \Gamma_j(ka)j(\gamma a)] \end{vmatrix}$$

and $W_2 =$

$$\begin{vmatrix} [\Gamma_h(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)h(\gamma b)] & [\Gamma_j(\gamma b)h(kb) - \frac{\gamma^2}{k^2}\Gamma_h(kb)j(\gamma b)] \\ [\Gamma_h(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)h(\gamma a)] & [\Gamma_j(\gamma a)j(ka) - \frac{\gamma^2}{k^2}\Gamma_j(ka)j(\gamma a)] \end{vmatrix}$$

The determinant D is given by $D = W_1 W_2$. Evaluating the determinants W_1 and W_2 , and rearranging terms to group together the functions containing the complex arguments $z=ya$ and $z=yb$ gives the result

$$\begin{aligned}
 W_1 = & j(ka)h(kb)[\Gamma_j(ya)\Gamma_h(yb) - \Gamma_j(yb)\Gamma_h(ya)]_1 + \\
 & + \Gamma_j(ka)h(kb)[h(ya)\Gamma_j(yb) - j(ya)\Gamma_h(yb)]_2 \\
 & + j(ka)\Gamma_h(kb)[\Gamma_h(ya)j(yb) - \Gamma_j(ya)h(yb)]_3 \\
 & + \Gamma_j(ka)\Gamma_h(kb)[j(ya)h(yb) - h(ya)j(yb)]_4, \quad (205)
 \end{aligned}$$

and

$$\begin{aligned}
 W_2 = & j(ka)h(kb)[\Gamma_j(ya)\Gamma_h(yb) - \Gamma_h(ya)\Gamma_j(yb)]_1 + \\
 & + \Gamma_j(ka)h(kb)\frac{Y^2}{K^2}[h(ya)\Gamma_j(yb) - j(ya)\Gamma_h(yb)]_2 \\
 & + j(ka)\Gamma_h(kb)\frac{Y^2}{K^2}[\Gamma_h(ya)j(yb) - \Gamma_j(ya)h(yb)]_3 \\
 & + \Gamma_j(ka)\Gamma_h(kb)\frac{Y^4}{K^4}[j(ya)h(yb) - h(ya)j(yb)]_4. \quad (206)
 \end{aligned}$$

The bracketed terms have been numbered from 1 to 4, for further reference. Now it is possible to solve for the coefficients c and d to obtain p . Using Kramer's rule, the coefficient c is given by $c = U_1/D$, where

$$U_1 = \begin{vmatrix} \frac{-1}{Kb} & [\Gamma_j(yb)h(kb) - \Gamma_h(kb)j(yb)] & 0 & 0 \\ 0 & [\Gamma_j(ya)j(ka) - \Gamma_j(ka)j(ya)] & 0 & 0 \\ \frac{Y}{K} & \frac{-1}{Kb} & 0 & 0 \\ 0 & 0 & 0 & W_2 \end{vmatrix};$$

$$\text{thus, } c = \frac{-1}{kb} [\Gamma_j(\gamma a)j(ka) - \Gamma_j(ka)j(\gamma a)] w_2. \quad (207)$$

Similarly, solving for d, $d = U_2/D$, where

$$U_2 = \begin{vmatrix} [\Gamma_h(\gamma b)h(kb) - \Gamma_h(kb)h(\gamma b)] & \frac{-1}{kb} & 0 & 0 \\ [\Gamma_j(\gamma a)j(ka) - \Gamma_j(ka)j(\gamma a)] & 0 & 0 & 0 \\ 0 & \frac{\gamma}{k} \frac{-1}{kb} & & \\ 0 & 0 & & w_2 \end{vmatrix};$$

$$\text{thus } d = \frac{+1}{kb} [\Gamma_h(\gamma a)j(ka) - \Gamma_j(ka)h(\gamma a)] w_2. \quad (208)$$

Now solve for p from equation 194. The solution for p is

$$p = \frac{+1}{kb} \rho [h'(\rho)j(\rho) - j'(\rho)h(\rho)]_{\rho=\gamma a} w_1^{-1} = \frac{+1}{kb} \frac{+1}{\gamma a} w_1^{-1}. \quad (209)$$

Next, solve for f and g to obtain q. The solution for f is given by $f = U_3/D$, where

$$U_3 = \begin{vmatrix} & \frac{-1}{kb} & & 0 \\ w_1 & 0 & & 0 \\ 0 & 0 & \frac{\gamma}{k} \frac{-1}{kb} & [\Gamma_j(\gamma b)h(kb) - \frac{\gamma^2}{k^2} \Gamma_h(kb)j(\gamma b)] \\ 0 & 0 & 0 & [\Gamma_j(\gamma a)j(ka) - \frac{\gamma^2}{k^2} \Gamma_j(ka)j(\gamma a)] \end{vmatrix};$$

$$\text{thus } f = \frac{\gamma}{k} \frac{-1}{kb} [\Gamma_j(\gamma a) j(ka) - \frac{\gamma^2}{k^2} \Gamma_j(ka) j(\gamma a)] w_1. \quad (210)$$

The solution for g is $g = U_4/D$, where

$$U_4 = \begin{vmatrix} & 0 & \frac{-1}{kb} \\ w_1 & 0 & 0 \\ 0 & 0 & [\Gamma_j(\gamma b) h(kb) - \frac{\gamma^2}{k^2} \Gamma_h(kb) j(\gamma b)] \frac{\gamma}{k} \frac{-1}{kb} \\ 0 & 0 & [\Gamma_j(\gamma a) j(ka) - \frac{\gamma^2}{k^2} \Gamma_j(ka) j(\gamma a)] & 0 \end{vmatrix};$$

$$\text{thus } g = \frac{\gamma}{k} \frac{+1}{kb} [\Gamma_h(\gamma a) j(ka) - \frac{\gamma^2}{k^2} \Gamma_j(ka) h(\gamma a)] w_1. \quad (211)$$

Now solve for q from equation 196. The solution for q is

$$q = \frac{\gamma^2}{k^2} \frac{+1}{kb} \frac{+1}{\gamma a} w_2^{-1}. \quad (212)$$

The solutions for the expansion coefficients which have been obtained in this section, and p and q in particular, are exact. Although the integer subscript n has been omitted ($p=p_n$, $q=q_n$, etc.), the results hold for all $n=1,2,3,\dots$. Thus, the exact solution for the electromagnetic fields for scattering from a spherical conducting shell have been obtained.

Next, the explicit solutions for p and q shall be used to obtain the shielding effectiveness of the sphere. Since the shielding effectiveness only depends on the cavity fields, the fields in the conducting walls and in the exterior region are not required. Consequently, explicit expressions for the coefficients a, b, c, d, f and g are not required in the analysis.

9. SHIELDING EFFECTIVENESS OF THE SPHERE

The shielding effectiveness shall be calculated with reference to the fields at the center of the sphere. The shielding effectiveness is most commonly defined as the ratio of the magnitude squared of the electric field at the center of the sphere to the electric field of the incident planewave,

$$SE_e = \left| \frac{E(r=0)}{E_{inc}} \right|^2. \quad (213)$$

This definition shall be called the electric shielding effectiveness. Similarly, the shielding effectiveness may be defined as the magnitude squared of the ratio of the magnetic field at the center of the sphere to the magnetic field of the incident wave,

$$SE_m = \left| \frac{H(r=0)}{H_{inc}} \right|^2. \quad (214)$$

This definition shall be called the magnetic shielding effectiveness. Another definition may be given in terms of the energy of the fields as the ratio of the energy density at the center of the sphere to the energy density of the incident wave

$$SE = 2 \frac{U(r=0)}{U_{inc}}. \quad (215)$$

This definition shall be called, simply, the electromagnetic shielding effectiveness. The factor of two is included so the shielding effectiveness goes to 1 as the frequency f goes to zero. It will be shown that for planewave incidence

$$SE = SE_e + SE_m. \quad (216)$$

In terms of the complex fields, and in free space, the energy density is given by the relation

$$U = \frac{1}{4}[\epsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu_0 \mathbf{H} \cdot \mathbf{H}^*]. \quad (217)$$

The energy density of the incident planewave is

$$U_0 = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}\mu_0 H_0^2. \quad (218)$$

To proceed with the calculation of the shielding effectiveness, first note that since $j_n(r=0) = 0$ for $n=1,2,3,\dots$, then

$$(M_{0,1n}^1)_{\rho=0} = 0, \quad n=1,2,3,\dots, \quad (219)$$

$$(M_{e,1n}^1)_{\rho=0} = 0, \quad n=1,2,3,\dots. \quad (220)$$

Also, from the identity

$$[\rho j_n(\rho)]' = \frac{\rho}{2n+1}[(n+1)j_{n-1}(\rho) - n j_{n+1}(\rho)], \quad n=1,2,3,\dots,$$

and the fact that

$$\lim_{\rho \rightarrow 0} \frac{j_n(\rho)}{\rho} = \begin{array}{ll} 1/3 & \text{for } n=1, \\ 0 & \text{for } n=2,3,4,\dots, \end{array}$$

the only nonzero wavefunction at $\rho=0$ of the type N is

$$(N_{0,11}^1)_{\rho=0} = \frac{2}{3} [\sin\theta \sin\phi \mathbf{e}_r - \cos\theta \sin\phi \mathbf{e}_\theta + \cos\phi \mathbf{e}_\phi], \quad (221)$$

$$(N_{e,11}^1)_{\rho=0} = -\frac{2}{3} [\cos\theta \cos\phi \mathbf{e}_\theta + \sin\phi \mathbf{e}_\phi]. \quad (222)$$

Therefore, the fields take on a particularly simple form at the center of the sphere since only one term in the series expansions is nonzero, the fields are given by

$$E = \frac{3}{2} E_0 q_1 N_{e,11}^1 |_{\rho=0} \quad (223)$$

$$H = \frac{3}{2} H_0 p_1 N_{o,11}^1 |_{\rho=0} \quad (224)$$

Therefore, the magnitudes of the electromagnetic fields at the origin are

$$|E|^2 = |E_0|^2 |q_1|^2 \left(\frac{3}{2}\right)^2 N_{e,11}^1 \cdot N_{e,11}^1 |_{\rho=0} = |E_0|^2 |q_1|^2, \quad (225)$$

$$|H|^2 = |H_0|^2 |p_1|^2 \left(\frac{3}{2}\right)^2 N_{o,11}^1 \cdot N_{o,11}^1 |_{\rho=0} = |H_0|^2 |p_1|^2. \quad (226)$$

The shielding effectiveness, then, is given by

$$SE_e = |q_1|^2, \quad (227)$$

$$SE_m = |p_1|^2, \quad (228)$$

$$SE = |p_1|^2 + |q_1|^2, \quad (229)$$

This establishes the relation between the three definitions of shielding effectiveness given by equation 216. Substituting for p and q from equations 209 and 212, the final formulas for the electric and magnetic shielding effectiveness take the form

$$SE_e^{-1/2} = \frac{1}{|q|} = |k/\gamma| (ka) (kb) |W_2|, \quad (230)$$

$$SE_m^{-1/2} = \frac{1}{|p|} = |\gamma a| (kb) |W_1|. \quad (231)$$

where W_2 and W_1 are given by equations 205 and 206, respectively. Similarly, the electromagnetic shielding effectiveness takes the form

$$SE = (|k/\gamma|(ka)(kb)|W_2|)^{-2} + (|\gamma a|(kb)|W_1|)^{-2} \quad (232)$$

In terms of dB, the shielding effectiveness takes the form

$$SE_e \text{ (dB)} = -20 \text{ Log}|q_1| = 20 \text{ Log}(|k/\gamma|(ka)(kb)|W_2|), \quad (233)$$

$$SE_m \text{ (dB)} = -20 \text{ Log}|p_1| = 20 \text{ Log}(|\gamma a|(kb)|W_1|), \quad (234)$$

and

$$\begin{aligned} SE \text{ (dB)} &= -10 \text{ Log}(|p_1|^2 + |q_1|^2) \\ &= -10 \text{ Log}((|k/\gamma|(ka)(kb)|W_2|)^{-2} + (|\gamma a|(kb)|W_1|)^{-2}). \end{aligned} \quad (235)$$

All these results are exact. The spherical Bessel and Hankel functions which are needed to calculate these quantities are given in Appendix F. Appendix F also gives various approximations for the spherical Bessel functions which are used in the numerical calculations. For the low-frequency calculations the quantities p_1 and q_1 are calculated from equations 209 and 212 by calculating W_1 and W_2 directly from equations 205 and 206. For the high-frequency calculations the brackets $[\]_1$, $[\]_2$, $[\]_3$, and $[\]_4$ which appear in equations 205 and 206 cannot be evaluated term by term for reasons explained in Appendix F. Therefore some approximations must be made to evaluate these quantities. The details of these approximations are given in Appendix F.

10. NUMERICAL CALCULATIONS FOR THE SPHERE

The shielding effectiveness is now calculated for a spherical shell with inner radius $a=77.4\text{mm}$ and outer radius $b=77.5\text{mm}$. The conductivity of the sphere is $\sigma=10^7\text{ohm}^{-1}\text{m}^{-1}$ which is representative of good conducting materials such as copper or aluminum. The results of the numerical calculations for the electromagnetic shielding effectiveness at low frequencies is shown in Figure 5. This plot covers the frequency range from 10^3 to 10^7 Hz. As the frequency approaches zero the shielding effectiveness goes to one, that is, zero dB, as expected. As the frequency goes to zero the magnetic shielding effectiveness SE_m approaches one since a static magnetic field is not shielded by the conductor. On the other hand the electric shielding effectiveness SE_e goes to zero since a static electric field cannot penetrate the conductor, that is, the electric field inside the cavity is zero. Therefore, the electromagnetic shielding effectiveness goes to 1 since $SE = SE_e + SE_m$.

It is remarkable that for frequencies from about 7×10^3 to 1×10^7 Hz the shielding effectiveness is practically a straight line. The slope of this part of the graph is 20 dB per octave. A straight line approximation is roughly

$$SE \text{ (dB)} = 20\log(f) + C,$$

where the point at which the straight line intersects $SE=0$ dB is $C = -20\log(f_0)$, where $f_0 = 5 \times 10^3$ Hz. Therefore the shielding effectiveness is approximately given by

$$SE \text{ (dB)} = 20\log(f/f_0), \quad (236)$$

or
$$SE = (f/f_0)^{-2}. \quad (237)$$

This is a very interesting result. According to equation 237 the shielding effectiveness goes as one over the frequency squared. Since the shielding effectiveness is equal to the square of the transfer function of the cavity fields with respect to the

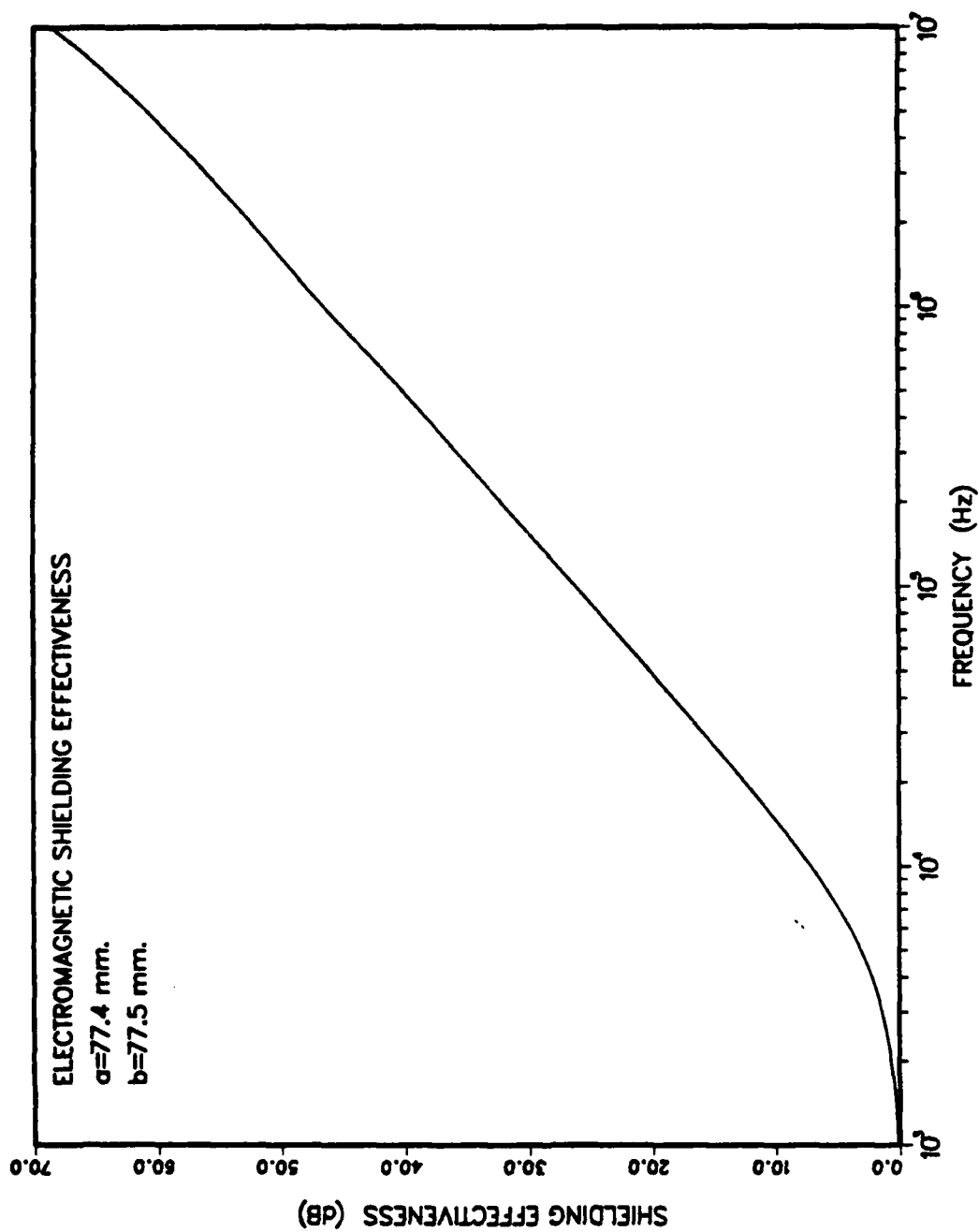


Figure 5. Shielding effectiveness for the sphere: low frequency results

incident fields this simple result suggests that analytical expressions may be obtainable in the case of an incident electromagnetic pulse. This idea will be developed momentarily. First the low frequency results for the electric field shielding effectiveness will be discussed. No graphical results are given for this case since they are not very interesting. The numerical results show that the electric shielding effectiveness decreases approximately as a straight line over at least seven orders of magnitude from 1 Hz to 10^7 Hz. The slope of the straight line is -20 dB per octave. Therefore, the electric field shielding effectiveness is given by the approximate equation

$$SE_e(\text{dB}) = -20\log(f) + C. \quad (238)$$

At $f=1$ Hz, the calculations give $SE_e=C= 283.8$ dB, and therefore

$$SE_e(\text{dB}) = -20\log(f/f_1), \quad (239)$$

or
$$SE_e = (f/f_1)^2, \quad (240)$$

where $f_1=1.549 \times 10^{14}$.

The approximate forms for the shielding effectiveness just obtained imply the relations

$$\left| \frac{H(0)}{H_{\text{inc}}} \right| = (\omega/\omega_0)^{-1}, \quad (241)$$

$$\left| \frac{E(0)}{E_{\text{inc}}} \right| = (\omega/\omega_1)^{+1}. \quad (242)$$

These simple transfer functions may be used to obtain an analytical expression for the fields at the origin when the incident planewave is a pulse. In cases when the frequency spectrum of the pulse is bandlimited to the range of validity of equations 241 and 242 the response may be obtained by using Laplace transforms. For example, in the case of a high altitude nuclear burst the resulting electromagnetic pulse will have a

waveform which is approximated very well by the double exponential

$$E_i(t) = E_0 \gamma (e^{-\beta t} - e^{-\alpha t}), \quad t > 0, \quad (243)$$

where $\alpha = 4.76 \times 10^8 \text{ s}^{-1}$ is the rise-constant, $\beta = 4 \times 10^6 \text{ s}^{-1}$ is the decay-constant, $E_0 = 50 \text{ kV/m}$ is the peak electric field, and $\gamma = 1.05$ is a normalization factor. For the sphere diameter chosen in this report the frequency spectrum of this pulse lies almost entirely within the range of validity of 241 and 242. Therefore, using 241 and 242, the attenuated fields at the center of the sphere will have the Laplace transforms

$$H(s) = \frac{A}{s} H_i(s), \quad (244)$$

$$E(s) = Bs E_i(s), \quad (245)$$

where $E_i(s)$ and $H_i(s)$ are the Laplace transforms of the incident pulse, $A = 2\pi f_0 = 3.14 \times 10^4$, and $B = 1/\omega_1 = 1.03 \times 10^{-15}$. From 243 the Laplace transform of the incident electromagnetic pulse is

$$F_i(s) = F_0 \gamma \frac{(\alpha - \beta)}{(s + \alpha)(s + \beta)}, \quad (246)$$

where F stands for either H or E . Therefore, taking the inverse transform the fields at the center of the sphere are found to be

$$H(t) = AH_0 \gamma \left[\frac{1}{\alpha} (1 + e^{-\alpha t}) - \frac{1}{\beta} (1 + e^{-\beta t}) \right], \quad (247)$$

$$E(t) = BE_0 \gamma [ae^{-\alpha t} - \beta e^{-\beta t}]. \quad (248)$$

The analytical solution for $E(t)$ is not of much practical value because the attenuation factor A is so great. Nonetheless, these expressions are a good approximation for the EMP response of a spherical shield of these dimensions, i.e., $a = 77.5 \text{ mm}$. As a final comment, note that it is not necessary to use Laplace transforms directly. Using the properties of the Laplace transform the expressions 244 and 245 imply that the response in the time

domain $H(t)$ and $E(t)$ are just the integral and the derivative of the respective inputs $H_i(t)$ and $E_i(t)$.

Consider next the high-frequency results for the spherical shield shown in Figure 6. These high frequency calculations were made by making certain approximations to the spherical Bessel and Hankel functions which are detailed in Appendix F. The results which are given in Figure 6, however, are entirely accurate. The most prominent feature in this Figure are the sharp dips above 10^9 Hz. These are resonances similar to those obtained for the cylindrical shield. The resonance condition for the spherical shield are derived in Appendix F. The resonances occur when either of the following conditions is met:

$$j_1(ka) = 0, \quad (249)$$

$$[\rho j_1(\rho)]'_{\rho=ka} = 0. \quad (250)$$

The first few zeros of equation 250 are listed in Appendix F. In general, the shielding effectiveness is reduced by three or four orders of magnitude at resonance. However this effect is negligible when compared to the overall magnitude of the attenuation at these frequencies. As in the case of the cylindrical shield an approximate formula for the shielding effectiveness is derived in Appendix F which has the form

$$SE = \frac{8\omega\epsilon_0}{\sigma} e^{-\sqrt{2\omega\sigma\mu_0}(b-a)}. \quad (251)$$

This approximate formula is plotted as a dashed curve in Figure 6. Note that this is the same formula obtained for the cylinder, equation 161, except for a factor of 2. A comparison between this formula and the exact solution shows that this is a very good approximation for frequencies in the resonance regime. As explained in Appendix F, this formula has a lower limit on its range of application since it diverges for lower frequencies.

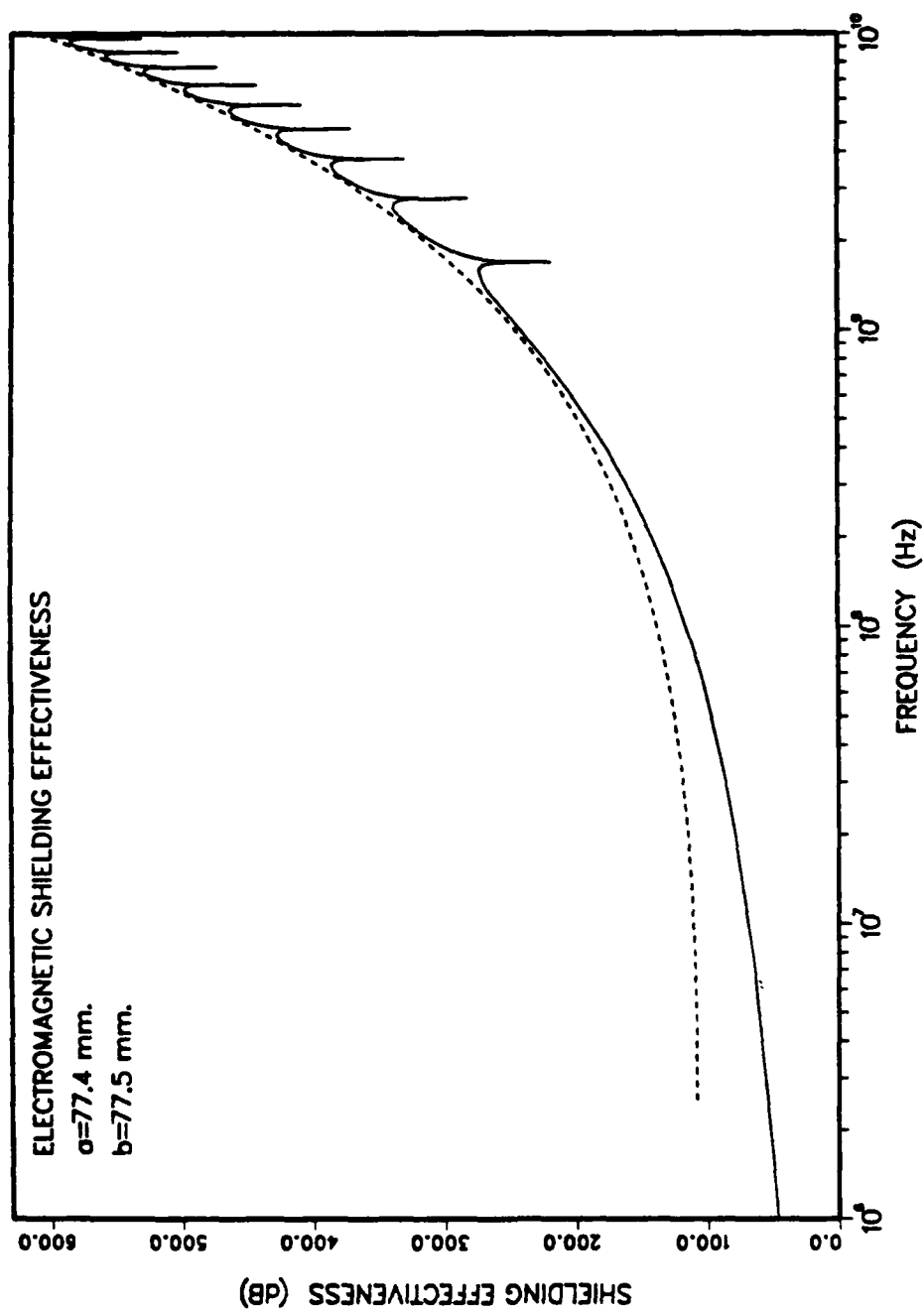


Figure 6. Shielding effectiveness for the sphere: high frequency results

CONCLUSIONS

The theory of electromagnetic shielding has been presented for a circular conducting cylinder and a conducting sphere. Starting from first principles, the complete solutions for the electromagnetic fields were developed first for the scattering of a planewave by a cylindrical conducting shell, and second for a spherical conducting shell. The shielding effectiveness of the cylinder was defined with respect to the fields at the center of the cylinder (on axis), and the shielding effectiveness of the sphere was defined with respect to the fields at the center of the sphere. In each case the fields at the origin have a particularly simple form since all the terms in the series expansion vanish except for the first one, and therefore it is possible to exhibit a single expression for the shielding effectiveness. This is desirable from a practical point of view since problems with convergence of infinite series may be avoided in the numerical calculations.

The most outstanding feature of the shielding effectiveness is the existence of resonances. In general, a series of discrete resonances occur for high frequencies where $ka \gg 1$. The electromagnetic resonances for the cylinder and the sphere have been investigated in detail and the resonance conditions have been derived for both structures. In addition, an asymptotic formula for the shielding effectiveness has been developed which appears to be a good approximation in the resonance regime. To this authors knowledge this result is new and it has not been published in the open literature. While this formula does not reproduce the resonances, it does account for the average magnitude of the shielding effectiveness in the high frequency limit.

Numerical calculations were performed for both the cylinder and the sphere, and the shielding effectiveness was plotted as a function of frequency for a shield with an outer diameter of 155 mm, a thickness of 0.1 mm, and a conductivity of $10^7 \text{ ohm}^{-1}\text{m}^{-1}$. The results are given in figures 3, 4, 5, and 6. As expected, the features of the shielding effectiveness are qualitatively and quantitatively similar for both the cylinder and the sphere. The magnitude of the frequency dependence of the shielding effectiveness is the same for both structures. Both structures show similar resonant behavior, although not at the same resonant frequencies. And, both structures have identical asymptotic forms for frequencies in the resonance regime. The almost linear increase of the shielding effectiveness at low

frequencies which was discovered for the sphere could not be demonstrated analytically. However, this is believed to be a general feature of the solutions for both the cylinder and the sphere.

APPENDIX A
NUMERICAL CALCULATION OF BESSEL FUNCTIONS

To calculate the series expansion coefficients it is necessary to evaluate Bessel and Hankel functions of both real and complex arguments. Specifically, it is necessary to evaluate $J_0(z)$, $J_1(z)$, $H_0(z)$, and $H_1(z)$, and the first derivatives of these functions, where z is in general complex. The Bessel function of the first kind is defined by the series

$$J_n(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!} \left(\frac{z}{2}\right)^{2k+n}, \quad n=0,1,2,\dots \quad (A-1)$$

The Neumann function, or Bessel function of the second kind, is defined here for positive integer indicies by

$$Y_n(z) = \frac{2}{\pi} [\gamma + \ln(\frac{z}{2})] J_n(z) - \frac{1}{\pi} \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!} \left(\frac{z}{2}\right)^{2k-n} + \\ - \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!} \left(\frac{z}{2}\right)^{2k+n} \left\{ 1 + \frac{1}{2} + \dots + \frac{1}{k} + 1 + \frac{1}{2} + \dots + \frac{1}{k+n} \right\}. \quad (A-2)$$

In the case $n=0$, the second term in this definition, the finite sum from $k=0$ to $n-1$ is omitted. This term is not present in the definition of $Y_0(z)$. For negative indicies the Bessel functions shall be defined by

$$J_{-n}(z) = (-1)^n J_n(z), \quad (A-3)$$

$$Y_{-n}(z) = (-1)^n Y_n(z). \quad (A-4)$$

The Hankel functions are defined in terms of the Bessel functions by the relations

$$H_n^1(z) = J_n(z) + iY_n(z), \quad (A-5)$$

$$H_n^2(z) = J_n(z) - iY_n(z). \quad (A-6)$$

The Bessel functions are calculated in two different ways depending on the magnitude of z . The series expansions A1 and A2 are used when $|z| \leq 10$, and the asymptotic series expansions are used when $|z| > 10$. Using the defining equation A1 the real and imaginary parts of the Bessel functions are

$$\operatorname{Re}[J_n(z)] = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!} \left(\frac{r}{2}\right)^{2k+n} \cos[(2k+n)\theta] , \quad (\text{A-7})$$

$$\operatorname{Im}[J_n(z)] = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!} \left(\frac{r}{2}\right)^{2k+n} \sin[(2k+n)\theta] , \quad (\text{A-8})$$

where $z = re^{i\theta}$. These series are used to calculate the Bessel functions $J_0(z)$ and $J_1(z)$ when $|z| \leq 10$. If the series is truncated after the 26th term, then the error is of the order of the first term neglected, roughly 10^{-16} for $|z| = 10$. For the Bessel function of the second kind the real and imaginary parts are given by

$$\operatorname{Re}[Y_0(z)] = \frac{2}{\pi} \{ \operatorname{Re}[J_0(z)](\gamma + \ln(r/2)) - \theta \operatorname{Im}[J_0(z)] - S_0 \} , \quad (\text{A-9})$$

$$\operatorname{Im}[Y_0(z)] = \frac{2}{\pi} \{ \operatorname{Im}[J_0(z)](\gamma + \ln(r/2)) + \theta \operatorname{Re}[J_0(z)] - T_0 \} , \quad (\text{A-10})$$

$$\operatorname{Re}[Y_1(z)] = \frac{2}{\pi} \{ \operatorname{Re}[J_1(z)](\gamma + \ln(r/2)) - \theta \operatorname{Im}[J_1(z)] - \frac{1}{r} \cos \theta - S_1 \} , \quad (\text{A-11})$$

$$\operatorname{Im}[Y_1(z)] = \frac{2}{\pi} \{ \operatorname{Im}[J_1(z)](\gamma + \ln(r/2)) + \theta \operatorname{Re}[J_1(z)] + \frac{1}{r} \sin \theta - T_1 \} , \quad (\text{A-12})$$

where $\gamma = 0.577215664902$ is Euler's constant; and S_0 , S_1 , T_0 , and T_1 are given by the following series:

$$S_0 = \sum_{k=1}^{\infty} \frac{(-1)^k r^{2k}}{2^{2k} (k!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right) \cos(2k\theta), \quad (A-13)$$

$$T_0 = \sum_{k=1}^{\infty} \frac{(-1)^k r^{2k}}{2^{2k} (k!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right) \sin(2k\theta), \quad (A-14)$$

$$S_1 = \sum_{k=0}^{\infty} \frac{(-1)^k r^{2k+1}}{2^{2k+2} k! (k+1)!} \left[2\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right) + \frac{1}{k+1}\right] \cos(2k+1)\theta, \quad (A-15)$$

$$T_1 = \sum_{k=0}^{\infty} \frac{(-1)^k r^{2k+1}}{2^{2k+2} k! (k+1)!} \left[2\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right) + \frac{1}{k+1}\right] \sin(2k+1)\theta. \quad (A-16)$$

To calculate $Y_0(z)$ and $Y_1(z)$ these series are truncated after the 26th term.

If the magnitude of z is greater than 10, $|z| > 10$, then the Bessel functions are calculated by using the asymptotic expansion

$$J_n(z) = \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} \{P_n(z) \cos[z - (2n+1)\frac{\pi}{4}] - Q_n(z) \sin[z - (2n+1)\frac{\pi}{4}]\}, \quad (A-17)$$

$$Y_n(z) = \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} \{P_n(z) \sin[z - (2n+1)\frac{\pi}{4}] + Q_n(z) \cos[z - (2n+1)\frac{\pi}{4}]\}; \quad (A-18)$$

where

$$P_n(z) = 1 - \frac{(4n^2-1)(4n^2-9)}{2!(8z)^2} + \frac{(4n^2-1)(4n^2-9)(4n^2-25)(4n^2-49)}{4!(8z)^4} - \dots, \quad (A-19)$$

$$Q_n(z) = \frac{(4n^2-1)}{8z} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3!(8z)^3} + \dots \quad (A-20)$$

The corresponding asymptotic expansion for the Hankel function of the first kind is

$$H_n^1(z) = \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} \{P_n(z) + iQ_n(z)\} e^{i[z - (n + \frac{1}{2})\frac{\pi}{2}]}. \quad (A-21)$$

When 4 terms are retained in equations A-19 and A-20 the error in the calculated values of the Bessel functions $J_0(z)$, $J_1(z)$, $Y_0(z)$, $Y_1(z)$, or the Hankel functions $H_0(z)$, or $H_1(z)$, is less than 1×10^{-7} for $|z| > 10$. The accuracy improves as $|z|$ increases.

The derivatives of the Bessel functions are calculated from the functions themselves by means of the following identities:

$$\frac{dJ_0}{dz} = -J_1(z), \quad \frac{dY_0}{dz} = -Y_1(z), \quad \frac{dH_0}{dz} = -H_1(z); \quad (A-22)$$

$$J_n'(z) = J_{n-1}(z) - \frac{n}{2} J_n(z), \quad n=1, 2, 3, \dots; \quad (A-23)$$

$$Y_n'(z) = Y_{n-1}(z) - \frac{n}{2} Y_n(z), \quad n=1, 2, 3, \dots; \quad (A-24)$$

$$H_n'(z) = H_{n-1}(z) - \frac{n}{2} H_n(z), \quad n=1, 2, 3, \dots; \quad (A-25)$$

where $H_n(z)$ stands for either $H_n^1(z)$ or $H_n^2(z)$, the Hankel function of the first or second kinds.

APPENDIX B
CALCULATION OF d_0

For the physical parameters considered in this study, and for the frequency range of interest, the Bessel functions may be evaluated for the complex arguments $z=ya$ and $z=yb$, by using the principle asymptotic forms for the Bessel and Hankel functions. The asymptotic form for the Bessel function is given by

$$\begin{aligned} J_n(z) &= \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} \cos\left[z - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right] , \\ &= \left(\frac{1}{2\pi z}\right)^{1/2} \left\{ e^{i\left[x - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right]} e^{-y} + e^{-i\left[x - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right]} e^{+y} \right\} ; \end{aligned} \quad (B-1)$$

and, for the Hankel function of the first kind by

$$\begin{aligned} H_n(z) &= \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} e^{i\left[z - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right]} , \\ &= \left(\frac{2}{\pi}\right)^{1/2} z^{-1/2} e^{i\left[x - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right]} e^{-y} ; \end{aligned} \quad (B-2)$$

where $z = x+iy$. A further approximation is helpful. Using equation 21, and considering the relevant physical parameters, that is, $f \geq 5 \times 10^4$ Hz, $\sigma = 10^7$ ohm $^{-1}$ m $^{-1}$, and $b > a \geq 7.5$ mm, yields

$$|\gamma| = (\omega \mu_0 \sigma)^{1/2} > 1985,$$

and
$$\text{Arg}(\gamma) = \frac{1}{2} \tan^{-1}\left(\frac{\sigma}{\epsilon_0 \omega}\right) \approx \frac{\pi}{4},$$

which together imply

$$y = 2^{-1/2} |ya| > 10.$$

Therefore, the first term in equation B-1 may be neglected compared to the second term. This yields the further approximation

$$J_n(z) = \left(\frac{1}{2\pi z}\right)^{1/2} e^{-i\left[x - \left(n + \frac{1}{2}\right)\frac{\pi}{2}\right]} e^{+y} . \quad (B-3)$$

The calculation of d_0 involves the calculation of a numerator and a denominator. The denominator is given by equation 41, with $n=0$. Evaluating the determinant yields

$$\begin{aligned} V = & \gamma H_0(kb) [\gamma J_0(ka) \{ J'_0(\gamma b) H'_0(\gamma a) - J'_0(\gamma a) H'_0(\gamma b) \}_1 + \\ & k J'_0(ka) \{ J_0(\gamma a) H'_0(\gamma b) - J_0(\gamma b) H'_0(\gamma a) \}_2] + \\ & -k H'_0(kb) [k J'_0(ka) \{ J_0(\gamma a) H_0(\gamma b) - J_0(\gamma b) H_0(\gamma a) \}_3 + \\ & + \gamma J_0(ka) \{ J_0(\gamma b) H'_0(\gamma a) - J'_0(\gamma a) H_0(\gamma b) \}_4] . \quad (B-4) \end{aligned}$$

The inner brackets have been numbered from 1 to 4. These inner brackets may be evaluated by first using the identities A-21 for the derivatives, and then substituting the required asymptotic forms B-2 and B-3. The evaluation of bracket number 1 according to this procedure goes as follows:

$$\{ \}_1 = J'_0(\gamma b) H'_0(\gamma a) - J'_0(\gamma a) H'_0(\gamma b) \quad (B-5)$$

$$= J_1(\gamma b) H_1(\gamma a) - J_1(\gamma a) H_1(\gamma b)$$

$$= A \zeta [e^{-i(x_2-x_1)} e^{(y_2-y_1)} - e^{i(x_2-x_1)} e^{-(y_2-y_1)}], \quad (B-6)$$

where

$$A = \frac{1}{\pi} |\gamma a|^{-1/2} |\gamma b|^{-1/2}, \quad \zeta = e^{-i\frac{\pi}{4}}, \quad (B-7)$$

$$\gamma a = x_1 + iy_1, \quad \text{and} \quad \gamma b = x_2 + iy_2.$$

Using the approximation: $\text{Arg}(\gamma) = \pi/4$, which is valid for all the physical parameters of interest, implies

$$x_2 - x_1 = y_2 - y_1 = \delta, \quad \text{where} \quad \delta = 2^{-1/2} |\gamma| (b-a). \quad (B-8)$$

Using this result, equation B-8, the first bracket, equation B-6, takes the following form

$$[]_1 = A\zeta\{e^{-i\delta}e^{\delta} - e^{i\delta}e^{-\delta}\} \quad (B-9)$$

$$= A\sqrt{2} \{ [\cos(\delta)\sinh(\delta) - \sin(\delta)\cosh(\delta)] + \\ - i[\cos(\delta)\sinh(\delta) + \sin(\delta)\cosh(\delta)] \} . \quad (B-10)$$

This latter form is useful for making calculations. Evaluating the next bracket, bracket number 2, in exactly the same way proceeds as follows:

$$[]_2 = J_0(\gamma a)H'_0(\gamma b) - J'_0(\gamma b)H_0(\gamma a) \quad (B-11)$$

$$= J_1(\gamma b)H_0(\gamma a) - J_0(\gamma a)H_1(\gamma b)$$

$$= A\zeta i\{e^{-i\delta}e^{\delta} + e^{i\delta}e^{-\delta}\} \quad (B-12)$$

$$= A\sqrt{2} \{ [\cos(\delta)\cosh(\delta) + \sin(\delta)\sinh(\delta)] + \\ + i[\cos(\delta)\cosh(\delta) - \sin(\delta)\sinh(\delta)] \} . \quad (B-13)$$

This completes the evaluation of bracket number 2. Proceeding similarly with bracket number 3:

$$[]_3 = J_0(\gamma a)H_0(\gamma b) - H_0(\gamma a)J_0(\gamma b) \quad (B-14)$$

$$= A\zeta\{e^{i\delta}e^{-\delta} - e^{-i\delta}e^{\delta}\} \quad (B-15)$$

$$= A\sqrt{2} \{ [\sin(\delta)\cosh(\delta) - \cos(\delta)\sinh(\delta)] + \\ + i[\cos(\delta)\sinh(\delta) + \sin(\delta)\cosh(\delta)] \} . \quad (B-16)$$

This completes the evaluation of bracket number 3. Lastly, the evaluation of bracket number 4 goes as follows:

$$[]_4 = J_0(\gamma b)H'_0(\gamma a) - J'_0(\gamma a)H_0(\gamma b) \quad (B-17)$$

$$= J_1(\gamma a)H_0(\gamma b) - H_1(\gamma a)J_0(\gamma b)$$

$$= A\zeta i\{e^{i\delta}e^{-\delta} + e^{-i\delta}e^{\delta}\} \quad (B-18)$$

$$= A\sqrt{2} \{ [\cos(\delta)\cosh(\delta) + \sin(\delta)\sinh(\delta)] + i[\cos(\delta)\cosh(\delta) - \sin(\delta)\sinh(\delta)] \}. \quad (B-19)$$

This completes the evaluation of bracket number 4.

Having evaluated each of the inner brackets 1, 2, 3, and 4 in the denominator, equation B-4, the following relations have been established:

$$[]_1 = -[]_3 = \lambda, \quad (B-20)$$

$$[]_2 = []_4 = \mu. \quad (B-21)$$

These equations shall define the two parameters λ and μ . Using these parameters, the denominator takes the following form

$$\begin{aligned} V &= \gamma H_0(kb) [\gamma J_0(ka)\lambda + kJ'_0(ka)\mu] + \\ &\quad kH'_0(kb) [kJ'_0(ka)\lambda - \gamma J_0(ka)\mu] \\ &= \lambda [\gamma^2 H_0(kb)J_0(ka) + k^2 H_1(kb)J_1(ka)] + \\ &\quad -\gamma k \mu [H_0(kb)J_1(ka) - H_1(kb)J_0(ka)] \end{aligned} \quad (B-22)$$

$$\text{or} \quad V = \lambda \alpha + \mu \beta \quad (B-23)$$

where

$$\alpha = \gamma^2 H_0(kb) J_0(ka) + k^2 H_1(kb) J_1(ka), \quad (B-24)$$

$$\beta = \gamma k [J_0(ka) H_1(kb) - H_0(kb) J_1(ka)]. \quad (B-25)$$

Equation B-23 is the final result for the denominator. The parameters α and β , given by equations B-24 and B-25, contain products of Bessel functions of real arguments. In general, these must be calculated by the methods described in Appendix A. The denominator is calculated by first calculating λ , μ , α , and β , from equations B-10, B-13, B-24, and B-25, respectively, and then using B-23.

The next element in the calculation of d_0 is the numerator, which is given by equation 40 with $n=0$. Evaluating the determinant gives $U_4 =$

$$\begin{aligned} & \gamma k [H_0(kb) J'_0(kb) - J_0(kb) H'_0(kb)] [J_0(\gamma a) H'_0(\gamma a) - H_0(\gamma a) J'_0(\gamma a)] \\ & = \gamma k [H_0(kb) J_1(kb) - J_0(kb) H_1(kb)] [J_0(\gamma a) H_1(\gamma a) - H_0(\gamma a) J_1(\gamma a)]; \end{aligned}$$

which, using the definition A-5, becomes

$$= \gamma k [J_1(kb) Y_0(kb) - J_0(kb) Y_1(kb)] [J_0(\gamma a) Y_1(\gamma a) - Y_0(\gamma a) J_1(\gamma a)].$$

The expressions in brackets may be simplified by using the identity

$$[J_{n+1}(z) Y_n(z) - J_n(z) Y_{n+1}(z)] = 2/\pi z.$$

Therefore, the numerator takes the final form

$$U_4 = \frac{4}{\pi^2 ab}. \quad (B-26)$$

This result is exact. The coefficient d_0 is the quotient of the numerator B-26 and the denominator B-23. In practice, however, it is easier to calculate the reciprocal of d_0 , which is given by

$$\frac{1}{d_0} = (\lambda\alpha + \mu\beta) \frac{\pi^2 ab}{4} . \quad (B-27)$$

It is important to specify the conditions under which this result is valid. Essentially, equation B-27 is a good approximation as long as the principal asymptotic forms are valid. Generally, the error in the principal asymptotic forms for the Bessel functions decreases for increasing $|z|$. For example,

when $|z| > 125$, the error is less than or equal to 3×10^{-3} ;

when $|z| > 1250$, the error is less than or equal to 3×10^{-4} ; and

when $|z| > 12500$, the error is less than or equal to 3×10^{-5} . It will be assumed that the results of this appendix are valid when $|z| > 125$. This choice for the validity criterion, while somewhat arbitrary, is substantiated by accurate calculations of $1/d_0$, which show that for all the cases studied in this report the error in B-27 is less than or equal to 10^{-5} when $|z| = 125$. For the arguments $z = \gamma a$ and $z = \gamma b$, this condition is satisfied when $|\gamma a| > 125$, which implies

$$|\gamma a| = \sqrt{\omega \mu_0 \sigma a} > 125 . \quad (B-28)$$

For the ranges of physical parameters considered in this appendix this criterion places a constraint on the minimum frequency f for which equation B-27 is applicable. For a given conductivity σ , and a given radius $r = a$, Table B-1 gives the minimum frequency obtained by equation B-28.

Finally, the resonance condition for the TM modes will be derived. It is known that electromagnetic resonance occurs for certain discrete frequencies, and that this causes a reduction in the shielding effectiveness at those frequencies. The shielding effectiveness may be reduced by 3 or 4 orders of magnitude due to this effect. By comparison with the resonance condition for a pair of parallel conducting planes separated by a distance $2a$, it is logical to expect a resonance to occur when the inner diameter of the cylinder is an integral number of half

Table B-1. Minimum frequency for a given cylinder radius.

a mm	$\sigma \Omega^{-1} \text{m}^{-1}$	f Hz
63.0	1×10^7	$\geq 5 \times 10^4$
44.4	1×10^7	$\geq 1 \times 10^5$
19.9	1×10^7	$\geq 5 \times 10^5$
14.0	1×10^7	$\geq 1 \times 10^6$
4.45	1×10^7	$\geq 1 \times 10^7$

wavelengths, that is, $2a = n(\lambda/2)$. Using the result for the shielding effectiveness, equation B-27, it may be shown that the resonance condition for a pair of concentric cylinders is $J_0(ka) = 0$. To prove this, first note that as the frequency increases from zero, the first resonance occurs approximately when $(\lambda/2) \sim 2a$, or $f \sim (c/4a)$. For such large frequencies, that is, $f > (c/4a)$,

$$\delta = \left(\frac{\omega \sigma \mu_0}{2} \right)^{1/2} (b-a) > \left(\frac{\pi c \sigma \mu_0 a}{4} \right)^{1/2} \Delta > 4.7, \quad (\text{B-29})$$

where it is assumed that $\Delta = (b-a)/a > 0.001$, $a > 7.5$ mm, and $\sigma > 10^7 \text{ ohm}^{-1} \text{m}^{-1}$. This frequency range will be called the resonance regime. If terms of the form $e^{-\delta}$ are neglected in the defining relations B-20 and B-21, for λ and μ , respectively, then this shows that $\mu = i\lambda$. Therefore, the inverse shielding effectiveness is approximately proportional to

$$|\lambda \alpha + \mu \beta| = |\lambda(\alpha + i\beta)| = |\lambda| |\alpha + i\beta|. \quad (\text{B-30})$$

Since $|\lambda| \sim e^{\delta}$ is an increasing function of frequency, the resonance phenomena must arise from the $\alpha + i\beta$ term. Studying this term in detail shows that since, for frequencies well below the infrared, $|y^2| \gg |y k| \gg |k^2|$, the dominant term in $\alpha + i\beta$ is the y^2 term:

$$\alpha + i\beta \sim \gamma^2 H_0(kb) J_0(ka). \quad (B-31)$$

Consequently, the minima in $\alpha + i\beta$ occur when $J_0(ka) = 0$. This establishes the resonance condition for the TM modes. Note that since the zeros of $J_0(x)$ and $J_1(x)$ alternate for x real, neither α or β is ever zero, and consequently the shielding effectiveness never goes to zero. The resonant frequencies are given by

$$ka = j_0(n), \quad n = 1, 2, 3, \dots; \quad (B-32)$$

$$\text{or} \quad f_n = \frac{c}{2\pi a} j_0(n), \quad n = 1, 2, 3, \dots; \quad (B-33)$$

where $j_0(n)$ are the n th zeros of $J_0(x)$.

APPENDIX C
CALCULATION OF D_0

The calculation of D_0 is similar to the calculation of d_0 given in Appendix B. The magnitude of D_0 is essentially the inverse shielding effectiveness in the TE case, as $|d_0|$ was in the TM case. D_0 is given by equation 54. By expanding the determinant in equation 59, the denominator is given by $Y=$

$$\begin{aligned} H_0(kb) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) \{ J_0(ka) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) [J'_0(\gamma b)H'_0(\gamma a) - J'_0(\gamma a)H'_0(\gamma b)]_1 + \\ - \frac{k}{i\omega\epsilon_0} J'_0(ka) [J_0(\gamma a)H'_0(\gamma b) - J'_0(\gamma b)H_0(\gamma a)]_2 \} + \\ - \frac{k}{i\omega\epsilon_0} H'_0(kb) \{ \frac{k}{i\omega\epsilon_0} J'_0(ka) [J_0(\gamma a)H_0(\gamma b) - J_0(\gamma b)H_0(\gamma a)]_3 + \\ J_0(ka) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) [J_0(\gamma b)H'_0(\gamma a) - J'_0(\gamma a)H_0(\gamma b)]_4 \}. \quad (C-1) \end{aligned}$$

The inner brackets are numbered 1,2,3, and 4 for easy reference. Note that these brackets are equivalent to the brackets 1,2,3, and 4 in Appendix B. Using the notation from Appendix B, the expression for the denominator C-1 may be written

$$\begin{aligned} H_0(kb) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) \{ J_0(ka) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) ()_1 - \frac{k}{i\omega\epsilon_0} J'_0(ka) ()_2 \} + \\ - \frac{k}{i\omega\epsilon_0} H'_0(kb) \{ \frac{k}{i\omega\epsilon_0} J'_0(ka) ()_3 - J_0(ka) \left(\frac{Y}{\sigma - i\omega\epsilon} \right) ()_4 \}. \quad (C-2) \end{aligned}$$

Using the principal asymptotic forms B-2 and B-3 to evaluate the inner brackets, exactly as was done in Appendix B, leads to

$$()_1 = -()_3 = \lambda, \quad (C-3)$$

$$()_2 = ()_4 = \mu; \quad (C-4)$$

which are identical to B-20 and B-21. The details of these calculations are given in Appendix B. Using these results the

denominator becomes $Y =$

$$\left(\frac{Y}{\sigma - i\omega\epsilon}\right) H_0(kb) \left\{ \left(\frac{Y}{\sigma - i\omega\epsilon}\right) \lambda J_0(ka) - \frac{k}{i\omega\epsilon_0} \mu J'_0(ka) \right\} + \frac{k}{i\omega\epsilon_0} H'_0(kb) \left\{ \left(\frac{Y}{\sigma - i\omega\epsilon}\right) \mu J_0(ka) + \frac{k}{i\omega\epsilon_0} \lambda J'_0(ka) \right\}, \quad (C-5)$$

which may be written

$$Y = \lambda \xi + \mu \eta, \quad (C-6)$$

where

$$\begin{aligned} \xi &= [p^2 H_0(kb) J_0(ka) + q^2 H'_0(kb) J'_0(ka)] \\ &= [p^2 H_0(kb) J_0(ka) + q^2 H_1(kb) J_1(ka)], \end{aligned} \quad (C-7)$$

$$\begin{aligned} \eta &= pq [J_0(ka) H'_0(kb) - J'_0(ka) H_0(kb)] \\ &= pq [J_1(ka) H_0(kb) - J_0(ka) H_1(kb)], \end{aligned} \quad (C-8)$$

and, where

$$p = \frac{Y}{\sigma - i\omega\epsilon}, \quad \text{and} \quad q = \frac{k}{i\omega\epsilon_0}. \quad (C-9)$$

This completes the evaluation of the denominator. The evaluation of the numerator, which is given by the determinant in equation 58, proceeds exactly as in Appendix B. The numerator may be calculated exactly with the result

$$-pq \frac{2}{\pi kb} (1) \frac{2}{\pi ya} (-1) = -\frac{pq}{\gamma k} \frac{4}{\pi^2 ab} = \frac{-1}{i\omega\epsilon_0 (\sigma - i\omega\epsilon)} \frac{4}{\pi^2 ab} \quad (C-10)$$

The coefficient D_0 is the quotient of the numerator and the denominator given by equations C-6 and C-10, respectively. In

practice it is more convenient to work with the inverse of D_0 . The inverse of D_0 takes the form

$$\frac{1}{D_0} = (\lambda\alpha' + \mu\beta') \frac{\pi^2 ab}{4}, \quad (C-11)$$

where

$$\alpha' = -\frac{\gamma k}{pq} \xi = -\gamma k[(p/q)H_0(kb)J_0(ka) - (q/p)H_1(kb)J_1(ka)] \quad (C-12)$$

$$\beta' = -\frac{\gamma k}{pq} \eta = -\gamma k[J_1(ka)H_0(kb) - J_0(ka)H_1(kb)]. \quad (C-13)$$

A further simplification may be made by noting that

$$(q/p) = \left(\frac{\sigma - i\omega\epsilon}{i\omega\epsilon_0}\right) \frac{k}{\gamma} = -\left(\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\omega\epsilon_0}\right) \frac{k}{\gamma} = -\frac{\gamma}{k}. \quad (C-14)$$

Therefore

$$\alpha' = k^2 H_0(kb)J_0(ka) + \gamma^2 H_1(kb)J_1(ka) \quad (C-15)$$

$$\beta' = \gamma k[J_0(ka)H_1(kb) - J_1(ka)H_0(kb)] \quad (C-16)$$

The result for D_0 , equation C-11, is identical to the result for d_0 , equation B-27, except that α' and β' in the former equation replace α and β in the latter equation.

As in Appendix B, it is important to specify the conditions under which this result is valid. The equation for D_0 , equation C-11, will be assumed valid for $|\gamma a| > 125$. Just as in Appendix B, for a given radius $r=a$ and a given conductivity σ , this imposes a restriction on the minimum frequency for which C-11 is valid. This leads again to the inequality $\sqrt{\omega\sigma\mu_0}a > 125$. and the corresponding Table, Table B-1.

Lastly, the resonance condition for the TE modes will be derived. As in the TM case, electromagnetic resonance will occur for certain discrete frequencies, causing a reduction in the shielding effectiveness at those frequencies. As in the TM case, the shielding effectiveness may be reduced by 3 or 4 orders of

magnitude due to the resonance effect. Using the result for the shielding effectiveness, equation C-11, it may be shown that the resonance condition is $J_1(ka) = 0$. The derivation of this result is exactly the same as for the TM case in Appendix B. Based on the resonance condition for a pair of parallel conducting planes it is known that the first resonance occurs approximately when $(\lambda/2) \sim 2a$, or $f \sim (c/4a)$. For large frequencies such that $f \gg (c/4a)$, it may be assumed that $\delta \gg 4.7$, as was done in Appendix B, equation B-29. Therefore, in the resonance regime, terms of the form $e^{-\delta}$ may be neglected in the expressions for both λ and μ . This leads to the result $\mu = i\lambda$. Therefore the inverse shielding effectiveness is proportional to

$$|\lambda\alpha' + \mu\beta'| = |\lambda(\alpha' + i\beta')| = |\lambda| |\alpha' + i\beta'|. \quad (C-17)$$

Noting that $|\lambda| \sim e^{\delta}$ is an increasing function of frequency, the resonance phenomena must arise from the $\alpha' + i\beta'$ term. Since, for frequencies well below the infrared, $|\gamma^2| \gg |\gamma k| \gg |k^2|$, the dominant term in $\alpha' + i\beta'$ is the γ^2 term:

$$\alpha' + i\beta' \sim \gamma^2 H_1(kb) J_1(ka). \quad (C-18)$$

Consequently, the minima in $\alpha' + i\beta'$ occur when $J_1(ka) = 0$. This establishes the resonant condition for the TE modes. Note that since the zeros of $J_0(x)$ and $J_1(x)$ alternate for x real, neither α' or β' is ever zero, and consequently the shielding effectiveness is never zero. The resonant frequencies are given by

$$ka = j_{1,n}, \quad n = 1, 2, 3, \dots; \quad (C-19)$$

$$\text{or} \quad f_n = \frac{c}{2\pi a} j_{1,n}, \quad n = 1, 2, 3, \dots; \quad (C-20)$$

where $j_{1,n}$ are the n th zeros of $J_1(x)$.

APPENDIX D
APPROXIMATE FORMULAS FOR SHIELDING EFFECTIVENESS

It is extremely useful to have an approximate closed form result for the shielding effectiveness. Such a form is easily derived in the TM case by using the results of Appendix B. The shielding effectiveness, given by equation B-27, is

$$SE^{-1/2} = \frac{\pi^2 ab}{4} |\lambda\alpha + \mu\beta|, \quad (D-1)$$

which, for frequencies in the resonance regime, may be written

$$SE^{-1/2} = \frac{\pi^2 ab}{4} |\lambda| |\alpha + i\beta|. \quad (D-2)$$

Now substitute the result from Appendix B,

$$|\lambda| = \frac{1}{\pi} |\gamma|^{-1} (ab)^{-1/2} e^{\delta}, \quad (D-3)$$

which is valid in the resonance regime; and based on the result B-31, make the approximation

$$|\alpha + i\beta| \sim |\gamma^2| M_0(kb)M_0(ka) \sim |\gamma^2| \left(\frac{2}{\pi kb}\right)^{1/2} \left(\frac{2}{\pi ka}\right)^{1/2}, \quad (D-4)$$

where, for x real,

$$M_0(x) = |H_0(x)| = [J_0^2(x) + Y_0^2(x)]^{1/2}, \quad (D-5)$$

and where it is assumed that

$$|J_0(x)| \sim M_0(x). \quad (D-6)$$

This yields the simple formula

$$SE^{-1/2} = \frac{1}{2} \left(\frac{\sigma}{\omega \epsilon_0}\right)^{1/2} e^{\delta}, \quad (D-7)$$

or,

$$SE = \frac{4\omega\epsilon_0}{\sigma} e^{-2\delta}, \quad (D-8)$$

where

$$\delta = \left(\frac{\omega\sigma\mu_0}{2}\right)^{1/2} (b-a). \quad (D-9)$$

This shows that the shielding effectiveness is independent of the radius of the cylinder and depends only on the thickness of the shield. A formula for the shielding effectiveness in dB is:

$$SE_{dB} = 10\left\{\log\left(\frac{\sigma}{4\epsilon_0}\right) - \log(\omega) + (2\sigma\mu_0)^{1/2} (b-a) \log(e) \omega^{1/2}\right\}. \quad (D-10)$$

This formula is a good approximation in the resonance regime, that is, $f > (c/4a)$. However, the approximation breaks down for low frequencies. If equation D-8 is considered as a function of ω , that is, $f(\omega)$, then this function has a minimum at the frequency

$$\omega_0 = \frac{2}{\mu_0\sigma} \frac{1}{(b-a)^2}. \quad (D-11)$$

As ω goes to zero, $f(\omega)$ approaches infinity. Therefore, the formulas D-8 or D-10 should not be applied for frequencies less than ω_0 . The error at this frequency is roughly 38% for the cases studied in this report. Consequently, this formula only provides a rough estimate in this lower frequency range.

The same formulas for the shielding effectiveness may be derived for the TE case by using the results of Appendix C. The shielding effectiveness, given by equation C-11, is

$$SE^{-1/2} = \frac{\pi^2 ab}{4} |\lambda\alpha' + \mu\beta'|, \quad (D-12)$$

which, for frequencies in the resonance regime, may be written

$$SE^{-1/2} = \frac{\pi^2 ab}{4} |\lambda| |\alpha' + i\beta'| . \quad (D-13)$$

Now make the approximations given by D-3, D-4, D-5, and D-6. This again yields the formulas given in equations D-8 and D-10.

APPENDIX E

AN ALTERNATE DEFINITION OF SHIELDING EFFECTIVENESS

An alternate definition of shielding effectiveness may be given in terms of the energy density of the incident and axial fields. The most natural definition of shielding effectiveness is

$$SE = (2) \frac{U^{axis}}{U^{inc}}, \quad (E-1)$$

where U is the energy density of the fields. The factor of two is included so that the shielding effectiveness approaches 1 as the frequency approaches zero. The average energy density of an electromagnetic field is defined by the relation

$$U = \frac{1}{2} \{ \mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B} \}, \quad (E-2)$$

where \mathbf{E} , \mathbf{D} , \mathbf{H} , and \mathbf{B} are the real fields, and $\{\}$ denotes a time average. In terms of the complex fields, and in free space, this is equivalent to

$$U = \frac{1}{4} [\epsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu_0 \mathbf{H} \cdot \mathbf{H}^*]. \quad (E-3)$$

It is interesting to compare the definition E-1 to the definition used in Section 7 for the TM and TE modes, equations 140 and 141. It will be shown that these definitions are equivalent. The energy density of the incident planewave is

$$U^{inc} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{1}{2} \mu_0 H_0^2. \quad (E-4)$$

For the TM case the \mathbf{E} -field at $r=0$ is given by the $n=0$ term in the series expansion since $J_n(0)=0$ for $n=\pm 1, \pm 2, \dots$. The ϕ -component of the \mathbf{H} -field is given by the $n=\pm 1$ terms since $J'_n(0)=0$ otherwise. The energy density of the field on the axis of the cylinder is

$$(TM) \quad U^{axis} = \frac{1}{4} \epsilon_0 E_0^2 [|d_0|^2 + |d_{-1} + d_{+1}|^2]. \quad (E-5)$$

Similarly, for the TE case the H-field at $r=0$ is given by the $n=0$ term in the series, and the ϕ -component of the E-field is given by the $n = \pm 1$ terms. Therefore the energy density on the cylinder axis is

$$(TE) \quad U^{axis} = \frac{1}{4} \mu_0 H_0^2 [|D_0|^2 + |D_{-1} + D_{+1}|^2] . \quad (E-6)$$

The symmetry about the x-z plane implies that the fields are represented by a cosine series. This implies that $d_{-1} = -d_1$, and $D_{-1} = -D_1$. Therefore, the shielding effectiveness for the TM and TE modes are given by

$$(TM) \quad SE = |d_0|^2 , \quad (E-7)$$

$$(TE) \quad SE = |D_0|^2 . \quad (E-8)$$

In terms of dB:

$$(TM) \quad SE_{dB} = -20 \log |d_0| , \quad (E-9)$$

and

$$(TE) \quad SE_{dB} = -20 \log |D_0| . \quad (E-10)$$

These formula are equivalent to equations 146 and 147. Therefore the definitions 140 and 141 given in section 7 for the shielding effectiveness of the TM and TE modes, respectively, are equivalent to E-1.

Next, consider a perpendicularly incident wave with an arbitrary state of elliptical polarization. The definition E-1 lends itself naturally to such a situation. It will be shown that the approximate formula D-7 for the shielding effectiveness is true for any state of polarization of the incident wave. To begin, note that the TM incident wave has rectangular components

$$E_z = E_o \quad (E-11)$$

$$H_y = -H_o, \quad (E-12)$$

and the TE incident wave has the components

$$H_z = H_o \quad (E-13)$$

$$E_y = E_o. \quad (E-14)$$

where $E_o/H_o = \sqrt{\mu_o/\epsilon_o}$. The electromagnetic fields for an incident planewave having an arbitrary state of elliptical polarization may be written

$$c_1\{TM\}_0 + c_2\{TE\}_0 \quad (E-15)$$

where the bracket $\{\}_0$ stands for the totality of field components, that is, both E and H, for either the TM or TE incident waves. The coefficients c_1 and c_2 are complex constants such that

$$|c_1|^2 + |c_2|^2 = 1. \quad (E-16)$$

Using the explicit TM and TE field components given above, the incident wave may be written in the form

$$E_{inc} = E_o(c_1 e_z + c_2 e_y) \quad (E-17)$$

$$H_{inc} = H_o(c_2 e_z + c_1 e_y). \quad (E-18)$$

where e_y and e_z are the unit vectors in the y and z directions, respectively. The energy density of the incident wave is computed using equation E-3, which yields

$$U_{\text{inc}} = \frac{1}{2} \epsilon_0 |E_0|^2 (|c_1|^2 + |c_2|^2) = \frac{1}{2} \epsilon_0 |E_0|^2. \quad (\text{E-19})$$

Now energy density of the axial fields must be calculated. If the incident wave is of the form E-15 then by the principle of superposition the solutions for the total fields have the form

$$c_1 \{\text{TM}\} + c_2 \{\text{TE}\}, \quad (\text{E-20})$$

where inside the cylinder, for example, {TM} stands for the totality of field components, both E and H, given by equations 29, 30, and 31; and {TE} stands for the totality of field components given by equations 47, 48, and 49. Using this representation the fields on axis are given by

$$E_z = c_1 E_0 d_0, \quad (\text{E-21})$$

$$E_z = c_2 E_0 D_0. \quad (\text{E-22})$$

Consequently, the energy density on the axis of the cylinder is

$$U_{\text{axis}} = \frac{1}{4} \epsilon_0 |E_0|^2 (|c_1|^2 |d_0|^2 + |c_2|^2 |D_0|^2). \quad (\text{E-23})$$

Therefore, the shielding effectiveness for a perpendicularly incident planewave having an arbitrary state of elliptical polarization is

$$\text{SE} = |c_1|^2 |d_0|^2 + |c_2|^2 |D_0|^2. \quad (\text{E-24})$$

As expected, this is a superposition of the shielding effectiveness for the independent TM and TE modes. When the approximate formula D-8 of Appendix D is valid, then $|d_0| = |D_0|$ which implies that E-24 takes the form

$$\text{SE} = |d_0|^2 = |D_0|^2. \quad (\text{E-25})$$

This proves that the approximate formula D-8 is valid for any state of polarization of the incident wave.

APPENDIX F
APPROXIMATIONS FOR THE SPHERICAL BESSEL FUNCTIONS

As was shown in section 7, the fields at the origin are determined by the two coefficients p_1 and q_1 . Therefore the calculation of the shielding effectiveness is concerned primarily with the numerical evaluation of these two quantities. According to the equations for p_1 and q_1 , equations 209 and 212, the main problem is to evaluate the determinants W_1 and W_2 given by equations 205 and 206. To do this, special care must be taken when computing each of the four brackets $[\]_1$, $[\]_2$, $[\]_3$, and $[\]_4$.

By using the Rayleigh formulas, the spherical Bessel and Hankel functions and the required derivatives are given by

$$j_1(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}, \quad (F-1)$$

$$[zj_1(z)]' = (1 - \frac{1}{z^2})\sin z + \frac{1}{z}\cos z, \quad (F-2)$$

$$h_1(z) = -i(\frac{1}{z^2} - \frac{i}{z})e^{iz}, \quad (F-3)$$

$$[zh_1(z)]' = i(\frac{1}{z^2} - \frac{i}{z} - 1)e^{iz}. \quad (F-4)$$

In general, these formulas may be used to calculate any one of these quantities exactly. However, for the complex arguments $z=ya$ and $z=yb$, it is not practical to compute these quantities directly from the above equations, except at low frequencies. The reason is that for high frequencies the values of the complex arguments $z=ya$ and $z=yb$ yield values of the spherical Bessel function $j_1(z)$ which are astronomically large since they have a large positive exponent, and values of the Hankel function $h_1(z)$ which are astronomically small since they have a large negative exponent. The magnitudes of these numbers is much greater than the range of most computers; on the order of $10^{10,000}$. However, the Bessel and Hankel functions with complex arguments have been grouped together in the brackets $[\]_1$, $[\]_2$, $[\]_3$, and $[\]_4$, in equations 205 and 206, and since products of these functions appear in pairs the large positive and negative exponents cancel. By making a simple approximation the products of the Bessel and Hankel functions may be calculated quite easily and accurately. First note that for a good conductor, and for frequencies such

that $(\sigma/\omega\epsilon_0) \ll 1$, the propagation constant inside the conducting medium, γ , is given by

$$\gamma^2 = i\omega\mu_0(\sigma - i\omega\epsilon) = \frac{\omega^2}{k^2} \left(\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\omega\epsilon_0} \right) \approx \frac{\omega^2}{c^2} \left(i\frac{\sigma}{\omega\epsilon_0} \right), \quad (F-5)$$

which implies the approximation

$$\gamma \approx \frac{1}{c} \left(\frac{\omega\sigma}{\epsilon_0} \right)^{1/2} e^{i\frac{\pi}{4}} = (\omega\mu_0\sigma)^{1/2} e^{i\frac{\pi}{4}}. \quad (F-6)$$

Considering the following ranges of physical parameters:

$a > 7.5$ mm, $f > 5 \times 10^4$ Hz, and $\sigma > 10^7$ ohm $^{-1}$ m $^{-1}$, implies that the arguments of the spherical Bessel functions satisfy the following inequalities: $ka > 0.139$ and $|z| = |\gamma a| > 14.89$. Therefore, writing z in the form $z = \xi + i\xi$, these conditions imply $\xi > 10.0$. Consequently, if terms of the form $e^{iz} = e^{i\xi}e^{-\xi}$ are neglected compared to the terms of the form $e^{-iz} = e^{i\xi}e^{+\xi}$, then the formulas F-1 and F-2 may be approximated by the formulas

$$j_1(z) \approx \frac{1}{2} \left(\frac{1}{z} + \frac{1}{z} \right) e^{-iz}, \quad (F-7)$$

$$[zj_1(z)]' \approx -\frac{1}{2} \left(\frac{1}{z} + \frac{1}{z} - 1 \right) e^{-iz}. \quad (F-8)$$

These approximations shall be used to evaluate the brackets in equations 205 and 206. Proceeding with the evaluation of bracket number 1, let $z_1 = \gamma a$ and $z_2 = \gamma b$, then using F-2, F-4, F-7, and F-8 yields

$$[]_1 = \Gamma_j(z_1)\Gamma_h(z_2) - \Gamma_j(z_2)\Gamma_h(z_1) \quad (F-9)$$

$$\begin{aligned} &= [1 - z_1^{-2} - z_2^{-2} + (z_1 z_2)^{-1} + (z_1 z_2)^{-2}] i \sin(z_2 - z_1) + \\ &\quad - [z_1^{-1} - z_2^{-1} + z_1^{-2} z_2^{-1} - z_1^{-1} z_2^{-2}] i \cos(z_2 - z_1). \end{aligned} \quad (F-10)$$

Similarly, simplifying bracket number two yields

$$[]_2 = h_1(z_1)\Gamma_j(z_2) - j_1(z_1)\Gamma_h(z_2) \quad (F-11)$$

$$= -[z_1^{-2} - (z_1 z_2)^{-1} - (z_1 z_2)^{-2}] i \sin(z_2 - z_1) \\ - [z_1^{-1} + z_1^{-2} z_2^{-1} - z_1^{-1} z_2^{-2}] i \cos(z_2 - z_1). \quad (F-12)$$

Bracket number three is given by

$$[]_3 = \Gamma_h(z_1)j_1(z_2) - \Gamma_j(z_1)h_1(z_2) \quad (F-13)$$

$$= -[z_2^{-2} - (z_1 z_2)^{-1} - (z_1 z_2)^{-2}] i \sin(z_2 - z_1) + \\ + [z_2^{-1} - z_1^{-2} z_2^{-1} + z_1^{-1} z_2^{-2}] i \cos(z_2 - z_1). \quad (F-14)$$

And, bracket number 4 is given by

$$[]_4 = j_1(z_1)h_1(z_2) - h_1(z_1)j_1(z_2) \quad (F-15)$$

$$= [(z_1 z_2)^{-1} + (z_1 z_2)^{-2}] i \sin(z_2 - z_1) + \\ - [z_1^{-2} z_2^{-1} - z_1^{-1} z_2^{-2}] i \cos(z_2 - z_1). \quad (F-16)$$

In general, these approximations are very good for $|ya| > 14.14$. These formulas for each of the four brackets are used in the calculation of the shielding effectiveness of a spherical conducting shell at high frequencies. It is not necessary to separate the real and imaginary parts of these equations since all numerical work is performed on a computer which supports complex arithmetic.

In addition to the approximations F-7 and F-8, for the spherical Bessel functions, another set of approximations is

required when the magnitudes of the arguments is very small, that is, $|z| \ll 1$. Approximations of this type are needed to calculate the shielding effectiveness for very low frequencies where $ka \ll 1$. The problem at very low frequencies is not with the evaluation of the bracketed terms, but with the coefficients of these brackets in equations 205 and 206. These coefficients are products of spherical Bessel functions having the real arguments $x=ka$ and $x=kb$. If $x \ll 1$, problems arise if $j_1(x)$ and $[xj_1(x)]'$ are computed by using equations F-1 and F-2 on a calculator or computer which has an accuracy of 12 significant figures. For example, if $x=10^{-6}$, then on a machine with an accuracy of 12 significant figures, a simple algorithm based on F-1 and F-2 yields $j_1(x)=0$ and $[xj_1(x)]'=0$. This situation is not acceptable since it may lead to erroneous results for the shielding effectiveness. Therefore, for $|z| \ll 1$, the Bessel and Hankel functions shall be approximated by the relations

$$j_1(z) = \frac{1}{3}z, \quad (\text{F-17})$$

$$[zj_1(z)]' = \frac{2}{3}z, \quad (\text{F-18})$$

$$h_1(z) = \frac{1}{3}z - i\left(\frac{1}{2}z + \frac{1}{2}\right), \quad (\text{F-19})$$

$$[zh_1(z)]' = \frac{2}{3}z + i\left(\frac{1}{2}z - \frac{1}{2}\right). \quad (\text{F-20})$$

These approximations may be obtained from the defining relations F-1, F-2, F-3, and F-4 by straightforward series expansions of the trigonometric and exponential functions. These equations are utilized in numerical computations when $|z| < 10^{-3}$.

The resonance conditions shall now be derived for the electric and magnetic fields. By analogy with the interference effects in a Fabry-Perot interferometer it is expected that the fields inside the spherical cavity should exhibit resonances for certain discrete frequencies. Consider the resonances in a system of two parallel planes separated by a distance d . Resonances will occur when the separation of the planes is equal to an integral number of half-wavelengths, that is, $n(\lambda/2)=d$, or $f=(c/2d)n$. Therefore, it is logical to expect that the first resonance for the sphere will occur near the frequency $f=(c/4a)$, or equivalently

$ka = (\pi/2) \sim 1$. For frequencies such that $f \geq (c/4a)$, the magnitude of the arguments $z = \gamma a$ satisfies the inequality

$$|\gamma a| = (\omega \sigma \mu_0)^{1/2} a \geq \left(\frac{\pi a c \sigma \mu_0}{2} \right)^{1/2}. \quad (F-21)$$

For the parameters $\sigma = 10^7 \text{ ohm}^{-1} \text{ m}^{-1}$ and $a = 7.5 \text{ mm}$, this inequality yields $|\gamma a| > 6664$. Therefore, in the resonance regime, the approximations F-10, F-12, F-14, F-16 will certainly hold for the brackets $[\]_1$, $[\]_2$, $[\]_3$, and $[\]_4$.

The resonances for the magnetic field shall occur when the magnitude of the coefficient p_1 is a maximum, since at the center of the sphere $|H/H_0| = |p_1|$. This implies that the reciprocal of $|p_1|$ will be a minimum. To first order,

$$W_1 = j_1(ka)h_1(kb)[\]_1 \sim j_1(ka)h_1(kb) \sin[\gamma(b-a)], \quad (F-22)$$

according to F-10, F-12, F-14, and F-16. Therefore the resonances for the magnetic field will occur when the quantity

$$|p_1|^{-1} = |\gamma a|(kb)|W_1| \sim |\gamma a|(kb)|j_1(ka)||h_1(kb)||\sin[\gamma(b-a)]| \quad (F-23)$$

is a minimum. Clearly, this occurs when $j_1(ka) = 0$. This is the resonance condition for the magnetic field. Note that this is also the resonance condition for the magnetic shielding effectiveness SE_m , since $SE_m = |p_1|^2$.

The resonances for the electric field shall occur when the magnitude of the coefficient q_1 is a maximum. This implies that the reciprocal of $|q_1|$ will be a minimum. To first order,

$$\begin{aligned} W_2 &= \Gamma_j(ka)\Gamma_h(kb)\frac{\gamma^4}{k^4} [\]_4 \\ &\sim \Gamma_j(ka)\Gamma_h(kb)\frac{\gamma^4}{k^4} [(\gamma a)(\gamma b)]^{-1} \sin[\gamma(b-a)], \end{aligned} \quad (F-24)$$

according to F-10, F-12, F-14, and F-16. Therefore the resonances for the electric field will occur when the quantity

$$|q_1|^{-1} = |k/\gamma|(ka)(kb)|W_2| \\ = |\gamma/k||\Gamma_j(ka)||\Gamma_h(kb)||\sin[\gamma(b-a)]| \quad (F-25)$$

is a minimum. Clearly, this occurs when $\Gamma_j(ka)=0$, that is, $[\rho j_1(\rho)]'_{\rho=ka}=0$. This is the resonance condition for the electric field. This is also the resonance condition for the electric shielding effectiveness SE_e , since $SE_e = |q_1|^{-2}$.

For the electromagnetic shielding effectiveness, which is defined in terms of the energy density of the fields, the resonances will occur when either of the previously derived conditions are satisfied separately, since $SE = SE_m + SE_e$. Therefore, the resonance conditions are

$$j_1(ka) = 0, \quad (F-26)$$

$$[\rho j_1(\rho)]'_{\rho=ka} = 0. \quad (F-27)$$

The first few zeros of F-27 are given in Table F-1.

Lastly, an approximate formula for the SE of a sphere shall be derived which is identical to the approximate formula for the cylinder. This approximate formula shall be derived for frequencies in the resonance regime. For the magnetic shielding effectiveness, start with equation F-23:

$$|p_1|^{-1} = |\gamma a|(kb)|W_1| = |\gamma a|(kb)|j_1(ka)||h_1(kb)||\sin[\gamma(b-a)]| \quad (F-28)$$

Assuming that $|\gamma(b-a)| > 14.14$, the sine term may be approximated by a single exponential,

n	z_n
1	2.743707 269992
2	6.116764 264462
3	9.316615 628566
4	12.485937 368200
5	15.643866 106348
6	18.796253 353454

Table F-1. Zeros of $[zj_1(z)]'$ for z real, $z > 0$.

$$|\sin[\gamma(b-a)]| \sim \frac{1}{2} e^{|\gamma| 2^{-1/2}(b-a)}. \quad (F-29)$$

The magnitude of the spherical Bessel and Hankel functions may be roughly approximated by their modulus [[19], page 439, equations 10.1.28 and 10.1.29]

$$j_1(x) \sim \frac{1}{x} \left(1 + \frac{1}{x}\right),$$

$$h_1(x) \sim \frac{1}{x} \left(1 + \frac{1}{x}\right).$$

Assuming $ka \gg 1$, then to first order $j_1(ka) = (1/ka)$ and $h_1(ka) = (1/ka)$, which imply that equation F-28 takes the form

$$|p_1|^{-1} \sim |\gamma/k| \frac{1}{2} e^{|\gamma| 2^{-1/2}(b-a)}. \quad (F-30)$$

This yields the desired formula for the shielding effectiveness

$$SE_m = |p_1|^2 = \frac{4\omega\epsilon_0}{\sigma} e^{-\sqrt{2\omega\sigma\mu_0}(b-a)}. \quad (F-31)$$

Next, the same formula shall be derived for the electric shielding effectiveness. Start with equation F-25:

$$|q_1|^{-1} = |\gamma/k| |\Gamma_j(ka)| |\Gamma_h(kb)| |\sin[\gamma(b-a)]| \quad (F-32)$$

Assuming that $|\gamma(b-a)| > 14.14$, the sine term may again be approximated by a single exponential as in equation F-29. Consider the terms $\Gamma_j(ka)$ and $\Gamma_h(kb)$. Using a standard Bessel function identity, one may write

$$[\rho j_1(\rho)]' = \rho j_1'(\rho) + j_1(\rho) = \rho j_0(\rho) - j_1(\rho). \quad (F-33)$$

$$[\rho h_1(\rho)]' = \rho h_1'(\rho) + h_1(\rho) = \rho h_0(\rho) - h_1(\rho). \quad (F-34)$$

Then, roughly approximating $j_0(\rho)$ and $j_1(\rho)$ by their modulus, to first order, $1/\rho$, since $ka \gg 1$, this shows that

$$[\rho j_1(\rho)]' \sim 1 - \frac{1}{\rho} \sim 1, \quad (F-35)$$

$$[\rho h_1(\rho)]' \sim 1 - \frac{1}{\rho} \sim 1. \quad (F-36)$$

Employing these rough order of magnitude approximations one obtains

$$|q_1|^{-1} \sim |\gamma/k| \frac{1}{2} e^{|\gamma| 2^{-1/2}(b-a)}, \quad (F-37)$$

which again yields the formula

$$SE_e = |q_1|^2 = \frac{4\omega\epsilon_0}{\sigma} e^{-\sqrt{2\omega\sigma\mu_0}(b-a)}. \quad (F-38)$$

Therefore, the final formula for the electromagnetic shielding effectiveness in the resonance regime is

$$SE = SE_m + SE_e = \frac{8\omega\epsilon_0}{\sigma} e^{-\sqrt{2\omega\sigma\mu_0}(b-a)}. \quad (F-39)$$

This shows that the shielding effectiveness at the center of the sphere is independent of the radius of the sphere and depends only on the thickness and conductivity of the shield. In terms of dB, this formula may be written

$$SE \text{ (dB)} = 10\{\text{Log}(\sigma/8\epsilon_0) - \text{Log}(\omega) + \sqrt{2\sigma\mu_0}(b-a)\text{Log}(e) \omega^{1/2}\}. \quad (F-40)$$

In general, the formulas F-39 and F-40 are good approximations when the conditions $|ya| > 14.14$ and $|\gamma(b-a)| > 14.14$ are both satisfied simultaneously. This implies that $|ya| > 14.14$, or equivalently $(\omega\sigma\mu_0)^{1/2}a > 14.14$. These formulas break down for low frequencies and cannot be used for frequencies less than

$$\omega_0 = \frac{2}{\mu_0\sigma} \frac{1}{(b-a)^2}.$$

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